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Least squares inversion of self-potential (SP) data and application to the shallow flow of ground water in sinkholes

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[1] We propose a least squares inversion algorithm to determine the spatially variable depth of the water table in shallow unconfined aquifers using self-potential signals measured on the ground surface. Traditionally, the water table is determined only at few locations using piezometers. Our approach relates its shape with the distribution of the self-potential signals according to a Fredholm equation of the first kind. The latter is discretized to obtain a linear matrix formulation of the forward problem. This new formulation is very general and can account for the resistivity distribution of the vadose zone. It is used to setup the inverse problem using the approach of Tarantola (1987) for a test site located in Normandy (France) where 225 self-potential measurements were performed over an area of 15,400 m^2. Ground water flows through the loess overlying a low permeability clay-with-flint weathered chalk, at a depth between 1 to 7 meters, into sinkholes in chalk bedrock. The method determines the water table with a precision of 0.4 m. **Citation:** Jardani, A., A. Revil, F. Akoa, M. Schmutz, N. Florsch, and J. P. Dupont (2006), Least squares inversion of self-potential (SP) data and application to the shallow flow of ground water in sinkholes, *Geophys. Res. Lett.*, *33*, L19306, doi:10.1029/2006GL027458.

1. Introduction

[2] The traditional method used by hydrogeologists to determine the shape of the water table of an unconfined aquifer is to drill a set of wells. However, this method is costly and implies that information is often only available at a few locations. Hence, there is an increasing interest to develop non-intrusive geophysical methods to map the water table. The flow of ground water generates a detectable polarization of electrical charge in the ground, called the streaming potential [*Revil et al.*, 1999]. It can be observed at the ground surface through passive "self-potential" measurements of the electrical potential with non-polarizable electrodes [*Fournier*, 1989].

[3] We develop below an algorithm to retrieve the shape of the water table of an unconfined aquifer using self-potential data measured at the ground surface. Our analysis is built on the works of *Fournier* [1989] and *Revil et al.* [2004]. The effectiveness of this method is demonstrated with using field data from a test site in Normandy (France). The proposed method should also be applicable in the presence of heterogeneous resistivity distribution in the vadose zone.

2. Forward and Inverse Problem

[4] The electrical potential $\varphi(\mathbf{r})$ (in V) measured at the point P($\mathbf{r}$) located on the Earth's surface (Figure 1) is related to the hydraulic head h at point M(r') at the water table (in m) by a Fredholm equation of the first kind [*Fournier*, 1989; *Revil et al.*, 2004],

$$\varphi(\mathbf{r}) = \frac{c'}{2\pi} \int_{\partial\Omega} h(\mathbf{r}') \frac{(\mathbf{r}-\mathbf{r}')\cdot \mathbf{n}_S(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dS, \qquad (1)$$

where $\mathbf{n}_S$ is the unit outward vector normal to the water table $\partial\Omega$, dS describes a surface element of the water table, c' is an apparent streaming potential coupling coefficient (in mV m^{-1}). *Revil et al.* [2004] show that the apparent coupling coefficient entering Fournier's law can be written as $c' = \Theta\ C - C_s$ where C and C_s are the streaming potential coupling coefficients for the vadose zone and the aquifer, respectively. The parameter Θ is the ratio between the electrical conductivity of the saturated formation and of the vadose zone. A first-order approximation of equation (1) is $\varphi(P) = c'(h(P) - h(P_0))$ where P_0 is the reference station [*Revil et al.*, 2004]. This solution is used as the initial model in the inversion.

[5] The water table is discretized with a regular grid. The electrical potential $\varphi(P)$ measured at a given station P on the surface sums all the individual N-contributions to each elementary dipole:

$$\varphi(P) = \sum_{i=1}^{N} \varphi(h_i). \qquad (2)$$

Equation (2) is discretized to analyze the distribution of the self-potential due to a localized elementary source placed on the water table,

$$\varphi_i = \sum_{j=1}^{N} K_{ij} h_j, \qquad (3)$$

where K_{ij} is the kernel that corresponds to the influence of the element j over the measurement i. If the resistivity

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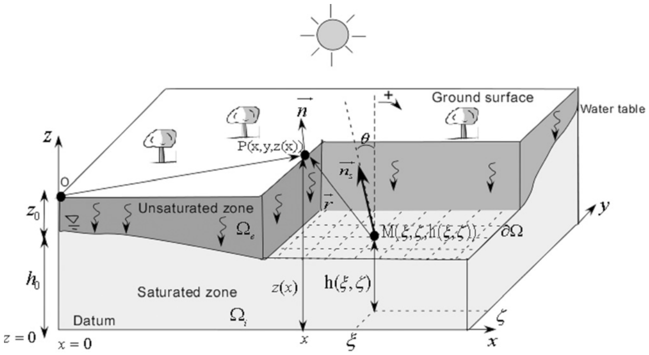

Figure 1. Sketch of the geometry of the water table corresponding to interface between an unconfined shallow aquifer (volume Ω_i) and the unsaturated (vadose) zone (volume Ω_e).

distribution is heterogeneous, the form of equation (1) is preserved but the kernel of the operator needs to be determined numerically to determine its value at each discretized cell of the ground.

[6] Let the observations for a set of n self potential data be represented by the vector $\mathbf{d} = (d_1, d_2, \ldots, d_n)$, and let the model response fitting data be the vector $\mathbf{f}(\mathbf{m})$ which is a function of p parameters being represented by the vector $\mathbf{m} = (m_1, m_2, \ldots, m_p)$ for the hydraulic heads. We consider a first-order Taylor expansion of the calculated model response and the model parameters $\mathbf{f}(\mathbf{m}) = \mathbf{f}(\mathbf{m}_0) + \mathbf{J}(\mathbf{m} - \mathbf{m}_0)$ where $\mathbf{J}$ is the Jacobian matrix depending of the geometry of the domain, $\mathbf{m}_0$ represents the initial model of parameters. We minimize the cost function $\mathbf{G} = (\mathbf{d} - \mathbf{f}(\mathbf{m}))^T \mathbf{C}_d^{-1} (\mathbf{d} - \mathbf{f}(\mathbf{m})) + (\mathbf{m} - \mathbf{m}_0)^T \mathbf{C}_m^{-1} (\mathbf{m} - \mathbf{m}_0)$, where $\mathbf{A}^T$ represent the transpose of matrix $\mathbf{A}$ and $\mathbf{d} - \mathbf{f}(\mathbf{m})$ the vector of errors between the calculated model response and data. The cost function $\mathbf{G}$ consists of the sum of a data objective function and a model objective function. The first contribution represents the data residuals weighted by the data error covariance matrix $\mathbf{C}_d$ while the second contribution measures the deviation of the parameters from an initial model $\mathbf{m}_0$, weighted by the parameter covariance matrix $\mathbf{C}_m$. Supposing that data errors are uncorrelated, C_m takes commonly the form of a diagonal matrix $\mathbf{C}_m = \sigma_m^2 \mathbf{I}_m$, where σ_m^2 is the estimated data error variance. For a stationary exponential field, the model covariance matrix is a full matrix correlating all model elements with each other and has the elements [*Tarantola*, 1987],

$$C_{ij} = C_0 \exp\left[-\sqrt{\left(\frac{x_i - x_j}{L_x}\right)^2 + \left(\frac{y_i - y_j}{L_y}\right)^2 + \left(\frac{z_i - z_j}{L_z}\right)^2}\right], \quad (4)$$

where L_i is the correlation length in the i-direction and C_0 is the amplitude of the variance. Minimization of $\mathbf{G}$ requires that $d\mathbf{G}/d\mathbf{m} = 0$, which yields:

$$\mathbf{m} = \mathbf{m}_0 + \mathbf{C}_d \mathbf{J}^T \left(\mathbf{J}\mathbf{C}_d\mathbf{J}^T + \mathbf{C}_m\right)^{-1} (\mathbf{d} - \mathbf{f}(\mathbf{m}_0)). \quad (5)$$

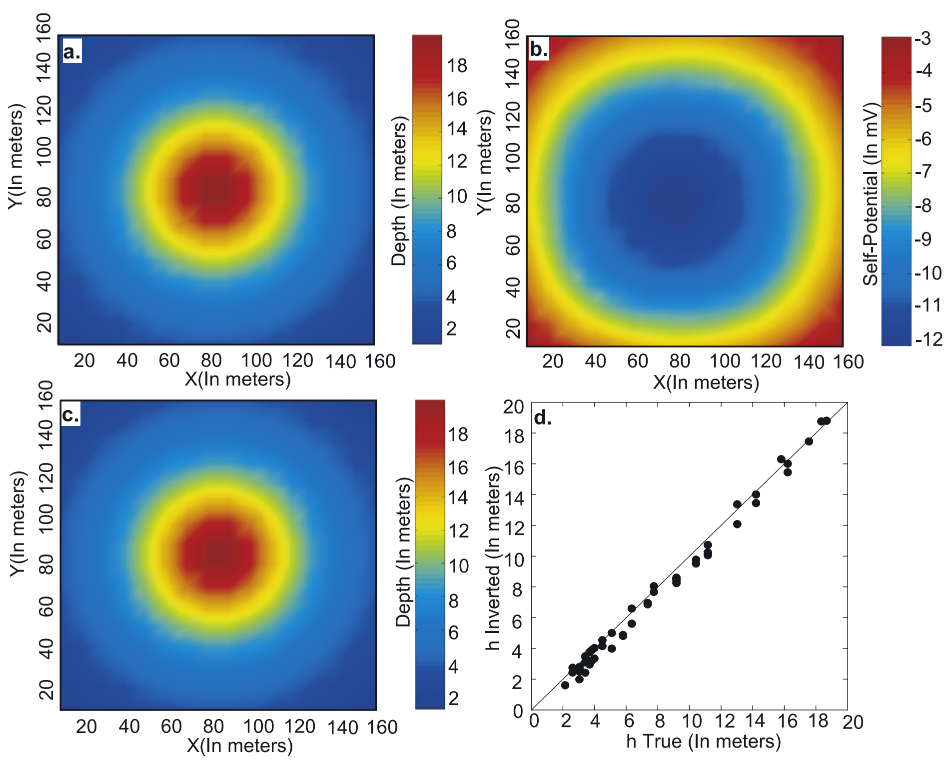

Figure 2. Synthetic test. (a) Synthetic case showing a Gaussian depression of the water table. (b) Distribution of the resulting self-potentials at the ground surface (with $c' = -4$ mV/m and $h_0 = 2$ m). (c) Inverted result of the water table (RMS error 1.15%). (d) True depth of the water table versus the inverted depth of the water table.

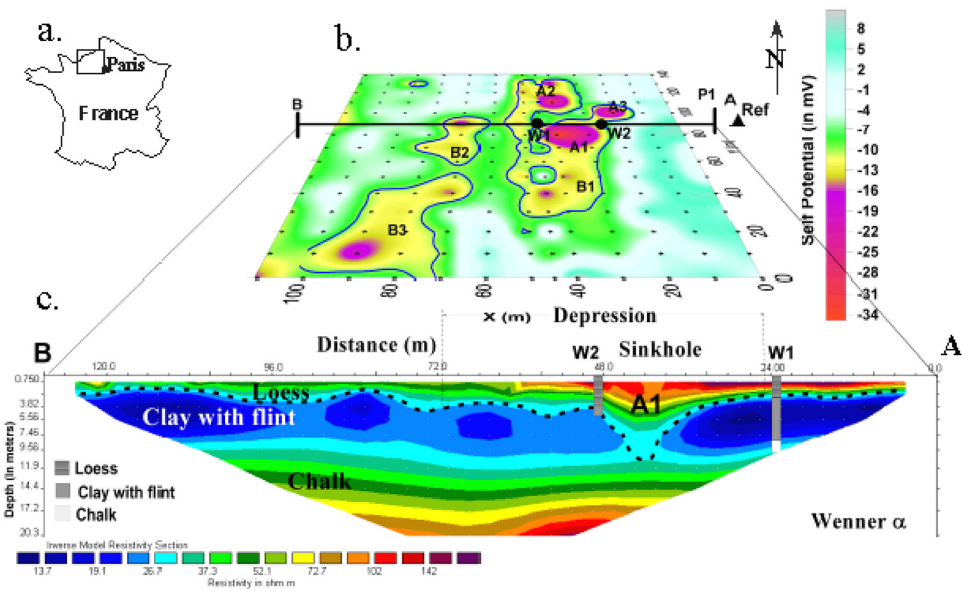

Figure 3. (a) Location of the test site in Normandy (France). (b) Self-potential map in Spring 2005. The anomalies A1 and A2 are sinkholes while A3, B2, and B3 are crypto-sinkholes. Ref represents the reference of the self-potential data and P1 is the resistivity profile. (c) Electrical resistivity tomogram inverted with RES2DINV [*Loke and Barker*, 1996] (Wenner-α array, electrode spacing 3 m, iteration 4, RMS error 1.4%).

Tarantola [1987] provides an equation to estimate the a posteriori covariance matrix of h.

[7] A synthetic Gaussian depression of the water table is used to test our algorithm. We determine first the resulting self-potential map by using equation (1). Then, we use equation (5) to recover the shape of the water table. The result, shown on Figure 2, provides a simple test of our algorithm.

3. Application

[8] The selected test site is located in Normandy (Figure 3a) and was recently investigated by *Jardani et al.* [2006] who presents a self-potential survey and electrical resistivity tomograms and performed numerical simulations of the forward coupled hydroelectric problem. *Jardani et al.* [2006] obtained 225 SP measurements were obtained in March 2005 with two $Cu/CuSO_4$ electrodes (see the dots on Figure 3b). The standard deviation on the measurements is 0.8 mV. The self-potential map realized in spring 2005 shows a set of negative self-potential anomalies (see the self-potential anomalies A1 and A2, Figure 3b). The self-potential anomalies A1 and A2 are two sinkholes, visible on the ground surface (Figure 3b). The anomaly A3 is likely a small sinkhole not yet associated with a depression of the topography of the ground surface. Boreholes (Figure 3c) indicate that the geology consists of a chalk bedrock with sinkholes covered by a loess layer exposed at the ground surface. A clay-with-flint layer corresponding to the weathered chalk is between the loess and the chalk bedrock (Figure 3c). The shape of the interface between the loess and clay-with-flint formations is well defined by the electrical resistivity tomogram (Figure 3c) and two boreholes. In March 2005, the piezometers indicate a small aquifer above this interface conducting flow to the sinkholes. Precipitation in Normandy peaks in this period. The depression of the water table above the sinkholes (Figure 3c) is largely due to the vertical infiltration of the water through the sinkholes

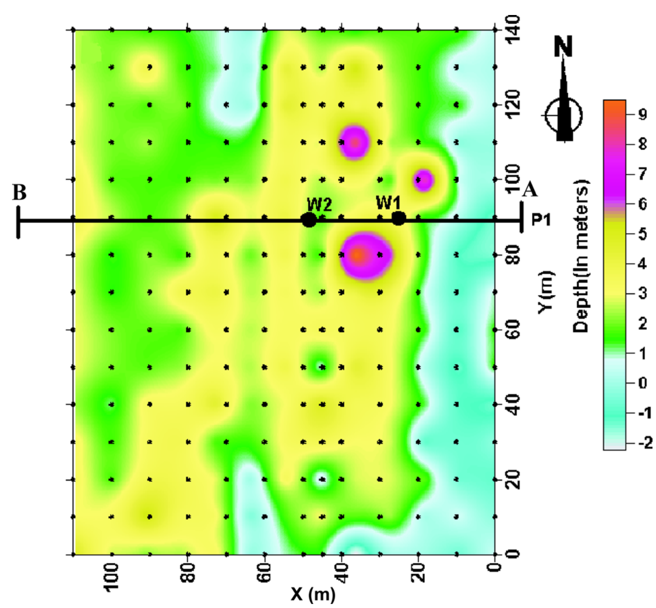

Figure 4. Inversion of the interface between the loess and the clay-with-flint formations. (RMS error 6.12%). The inversion shows clearly a wide depression of the interface with a North-South trend along which the sinkholes are aligned. The circular depressions of the water table correspond to the sinkholes.

but also reflects the depression of the clay-with-flint/loess interface at these locations.

[9] Laboratory experiments of electrical resistivity of the different formations and measurements of the streaming potential coupling coefficients [*Jardani et al.*, 2006] imply that $c' \sim -(4 \pm 2)$ mV m^{-1}. A borehole provides the depth of the loess/clay-with-flint interface, h_0 = 2 m, below the self-potential reference station (placed just outside the mapped area, Figure 4a). This puts an absolute depth constraint on our inversion, the result of which is shown on Figure 4. The mean uncertainty associated with the estimation of the depth of the water table is 0.4 m. An enhanced resolution self-potential survey was also performed along the resistivity profile (Figure 5a). The 2D inversion of these data is shown on Figure 5b and is consistent with the information of the electrical resistivity tomogram. To check the validity of the inversion, we plot the inverted results as a function of the depth of the clay-with-flint/loess interface determined from the electrical resistivity tomogram and the two boreholes W1 and W2 on Figure 3b. The final result is shown Figure 6. The inverted shape of the water table agrees reasonably well with the depth of the clay-with-flint/loess interface along which the ground water flows.

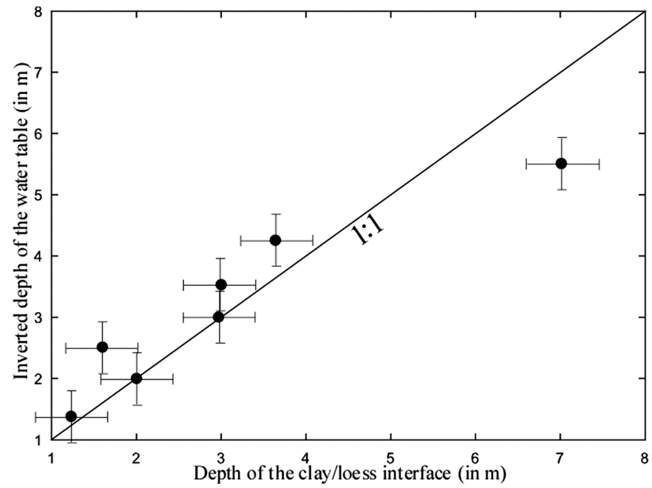

Figure 6. Inverted depth of the loess/clay-with-flint interface versus the height of the interface determined from the electrical resistivity data (r = 0.88). The error associated with the deepest point is coming from the small density of measurements performed over the sinkholes in the present study.

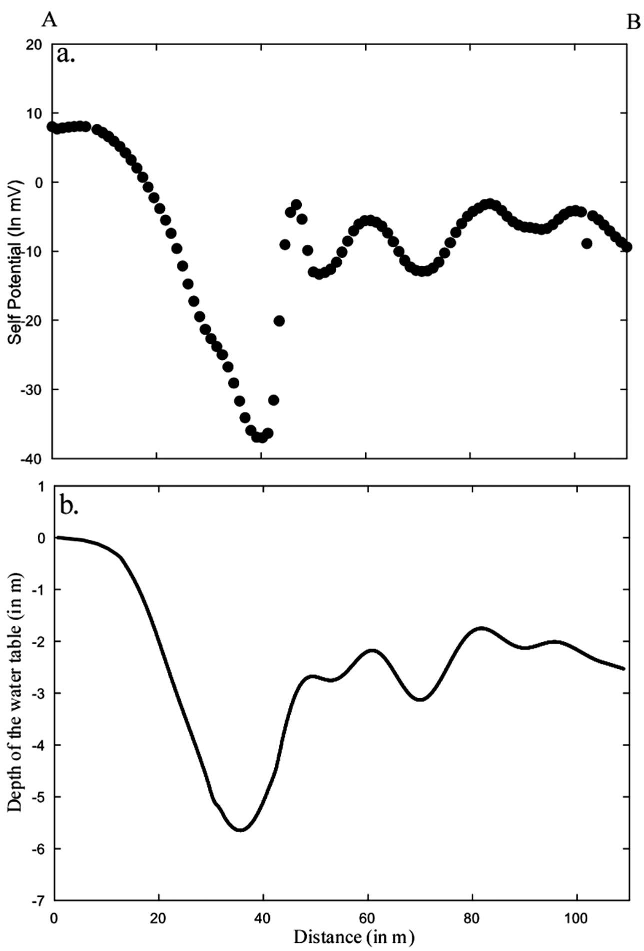

Figure 5. (a) High resolution self potential profile along the resistivity profile AB. (b) Inverted depth of the water table (RMS error 5.43%).

4. Conclusion

[10] We propose a new method to invert self-potential data in order to retrieve the shape of the water table. An application is made to a karstic area located in Normandy (France). We found a reasonable match between the position of the water table and that determined independently. This shows that self-potentials can be used quantitatively to determine the shape of the water table. However, the self-potential method is a passive method. Therefore there are other sources of signals that could potentially be misinterpreted as a depth change of the water table. To avoid this, we plan to add additional information in the inverse problem by performing a joint inversion of self-potential and EM-conductivity data as these methods offer complementary information.

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