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# A simple method to retrieve the complex eigenfrequency of the Earth's nearly diurnal-free wobble; application to the Strasbourg superconducting gravimeter data

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## SUMMARY

We have analysed more than four years of data from the Strasbourg superconducting gravimeter to retrieve the period and damping of the nearly diurnal-free wobble (NDFW). The removal of noise spikes is found to be crucial for an accurate determination of tidal-wave amplitudes and phases. A new simple algorithm is derived which allows an analytical solution for the NDFW period and damping using the complex gravimetric factors of three resonant diurnal waves. The results show a huge reduction of the confidence intervals when compared with a previous investigation from a Lacoste Romberg spring meter operated at the same station. Our results are in close agreement with values obtained from two other European superconducting gravimeters. The results are also compared with respect to values inferred from very long baseline interferometry (VLBI) measurements.

**Key words:** Earth's rotation, eigentheory, free oscillations, gravity, tides.



## INTRODUCTION

The nearly diurnal-free wobble (NDFW) is a retrograde motion of the Earth's rotation axis with respect to its figure axis with an eigenperiod close to 1 sidereal day. The associated motion in space is a long-period nutation called free core nutation (FCN). Because there are lunisolar tidal waves close in period to the NDFW, there is a resulting resonant enhancement of these waves. Similarly, the resonant behaviour of lunisolar nutations can be studied by very long baseline interferometry (VLBI) techniques. In this study, we focus on a gravity data set of slightly more than four years obtained from the French superconducting gravimeter located near Strasbourg (J9 station). After discussing some steps involved in the signal processing of tidal gravity, we develop a new method to derive analytically the parameters of the NDFW from the complex gravimetric factor of three resonant diurnal waves. The results for the period and damping of this eigenmode are then compared with other published results from superconducting gravimeters (SG) and VLBI.

## TIDAL ANALYSIS

Earth-tides analysis was initiated several decades ago when continuous tidal recording began around 1960 (see Melchior 1983). Basically, the observed tide is compared with the theoretical one (the amplitude, phase and frequency of tidal waves being very well determined from theory) in order to infer the Earth's response to tidal forcing. In gravimetry, most of the techniques involve a least-squares fitting of the gravimetric amplitude and phase factors ($\delta$, $\kappa$) for a given number of tidal groups. Each tidal group includes waves of close frequencies. The groups arise from the limitation in spectral resolution due to the limited duration of the observed signal. Hence, the model to represent the measured gravity signal mainly contains a tidal part which takes the form:

$$s_0(t) = \mathcal{R}e\left\{\sum_{k=1}^{n} \tilde{\delta}_k \sum_{j=1}^{m_k} A_{kj} \exp\left[i(\omega_{kj}t + \phi_{kj})\right]\right\}. \tag{1}$$

In this expression, $A_{kj}$ and $\phi_{kj}$ are respectively the theoretical amplitude and phase of the wave $j$ in the group $k$ ($m_k$ is the number of individual waves in a given group $k$) with angular frequency $\omega_{kj}$; $\tilde{\delta}_k = \delta_k \exp(i\kappa_k)$ is the complex gravimetric factor relative to the tidal group $k$.

In fact, the measured gravity signal is more complex and involves some other components which can be written:

$$s(t) = s_0(t) + b_1(t) + b_2(t), \tag{2}$$

where $b_1(t)$ is the sum of various effects always present in the observed signal-like instrumental drift, atmospheric loading, water-table changes, measurement noise, etc. The contribution of $b_1(t)$ can be reduced by applying appropriate models to the data (a typical example is to fit an exponential or a low-order polynomial function to get rid of the long-term drift of a superconducting gravimeter); however, $b_1(t)$ cannot be entirely removed. The $b_2(t)$ component consists of noise transient effects such as power failure, stormy weather and earthquakes. In the case of the superconducting gravimeters, a part of $b_2(t)$ is caused by liquid helium transfers to refill the instrument Dewar. The effects of such short disruptions on tidal analysis has been discussed by Florsch *et al.* (1991). In the least-squares analysis, the existence of a transient signal leads to an error when determining the parameters of any wave. However, one can estimate the error on the wave amplitude by assuming that it is close to the standard deviation of the spectral noise in the neighbourhood of the spectral peak under consideration. The expected error is then

$$\frac{A}{N\sqrt{2}},$$

where $N$ is the number of points in the time series and $A$ is the amplitude of the transient (see Florsch *et al.* 1991).

The process to reduce these various transient disturbances involves several steps. The original digitizing rate of the gravity signal is one value every 2 s and an analogue low-pass pre-filter (cut-off period of 50 s and 36 db per octave attenuation) is used to prevent aliasing. The signal is decimated to one point every 5 min by applying to the 2 s data a numerical low-pass symmetrical filter running over 20 min designed in a Strasbourg by R. Lecolazet (the result of convolving four averaging filters of different lengths). We first roughly correct gravity for local pressure using a constant coefficient of $-0.3\ \mu\text{gal mbar}^{-1}$ (e.g. Warburton & Goodkind 1977) representing the main part of the gravity-pressure admittance. After a visual inspection of the 5 min gravity samples, the damaged time intervals are eliminated and replaced by gaps. The duration of a gap ranges from 25 min to several hours. We 'fill' these gaps with a theoretically predicted tide, taking into account, for each major tidal group, a complex gravimetric factor which was obtained for our station from a previous tidal analysis.

Therefore, this prediction can be seen as the best, averaged over time, 'local' tide. After that, a low-pass decimation filter running over 24 hr and having a cut-off period of 3 hr is then applied to the 5 min values and provides hourly samples of gravity and local pressure. A first least-squares analysis is done using the HYCON (Hybrid least-squares frequency domain convolution) program (see Schüller 1986). Basically, it is a least-squares procedure fitting simultaneously lunisolar tides (based on the 505 wave expansion of Cartwright & Tayler 1971, and Cartwright & Edden 1973), local atmospheric pressure and polynomial drift. It is clear that the use of a more accurate tidal development based on recent tidal ephemeris (Tamura 1987; Xi Quiwen 1987, 1989; Merriam 1992a) as reference theoretical potential would lead to a better reduction of the tidal residual signal (which is the gravity signal obtained after subtracting the fitted local tide, the pressure-induced gravity and the long-term drift from the raw data) as shown by two recent studies (Wenzel & Zürn 1990; Hinderer, Crossley & Florsch 1991a).

A local pressure correction (complementary to the factor $-0.3\ \mu\text{gal (mbar}^{-1}$ already introduced) is included in the least-squares analysis. Indeed, the pressure signal is an additional input channel in the least-squares fit. Using a single correction coefficient means that, at time $t$, any increase in pressure decreases the gravity instantaneously by an amount given by the above coefficient. An alternative barometric admittance can be calculated with a convolution filter with a few coefficients. Currently three terms are considered and the gravity correction at time $t$ is then a function of pressure at times $t$, $t-1$, $t-2$ (assuming a unity sampling rate). In general, the dominant term of the filter coefficients is found to be at time $t$ while the two other coefficients decrease away (see e.g. Ducarme, Van Ruymbeke & Poitevin 1986), meaning that the filter asymmetry leads to some phase shift in the gravity response to pressure fluctuations. Even if the underlying physical mechanism is not obvious and may just reflect the lack of appreciation of the coherence spatial scale of the pressure systems (Merriam 1992b), such a filter can be seen as an effective admittance taking into account the gravity response to some atmospheric dynamics (e.g. the passage of a pressure front) rather than a static factor assuming a uniform pressure around the station. The Fourier transform of the convolution filter is frequency dependent while, in the case of a single coefficient, the transfer function is obviously constant in the frequency domain. From numerical tests, we were able to obtain a slightly better reduction in the average noise level of gravity residuals when applying a three-term convolution filter instead of a single coefficient to high-passed pressure and gravity records. However, the single or multicoefficient correction only involves local pressure effects. Further improvement may be obtained by considering regional or even global atmospheric pressure data and computing the resulting gravity load effect (Rabbel & Zschau 1985; Van Dam & Wahr 1987; Merriam 1992b).

Since the interpolation of the gaps corresponding to the damaged segments is not perfect, some transient signals still remain after the first least-squares fit, especially at the borders of the interpolation intervals. These weak signals are located in time on the gravity residuals, then isolated and removed from the original gravity signal (the input of the tidal analysis). This sequence is usually called 'despiking'. Afterwards a second and final tidal analysis of the despiked data is undertaken again using HYCON, Figs 1 and 2 illustrate these steps. The residual gravity signal obtained after the first least-squares analysis is plotted on the top part of Fig. 1. One can observe that there are large spikes with amplitudes of several $\mu$gal which are superimposed on the filtered residual noise. The histogram of the gravity residuals is then used to choose the threshold to remove these spikes; a typical value would be $3\sigma$, where $\sigma$ is the standard deviation of the distribution. By applying this threshold to the residual signal, we obtain the so-called despiking function, shown on the bottom part of Fig. 1. This function is zero most of the time except where the spikes having an amplitude larger than the fixed threshold are located. Then, the despiking function being subtracted from the raw gravity, a second least-squares procedure is performed, leading to the final residual plotted on Fig. 2.

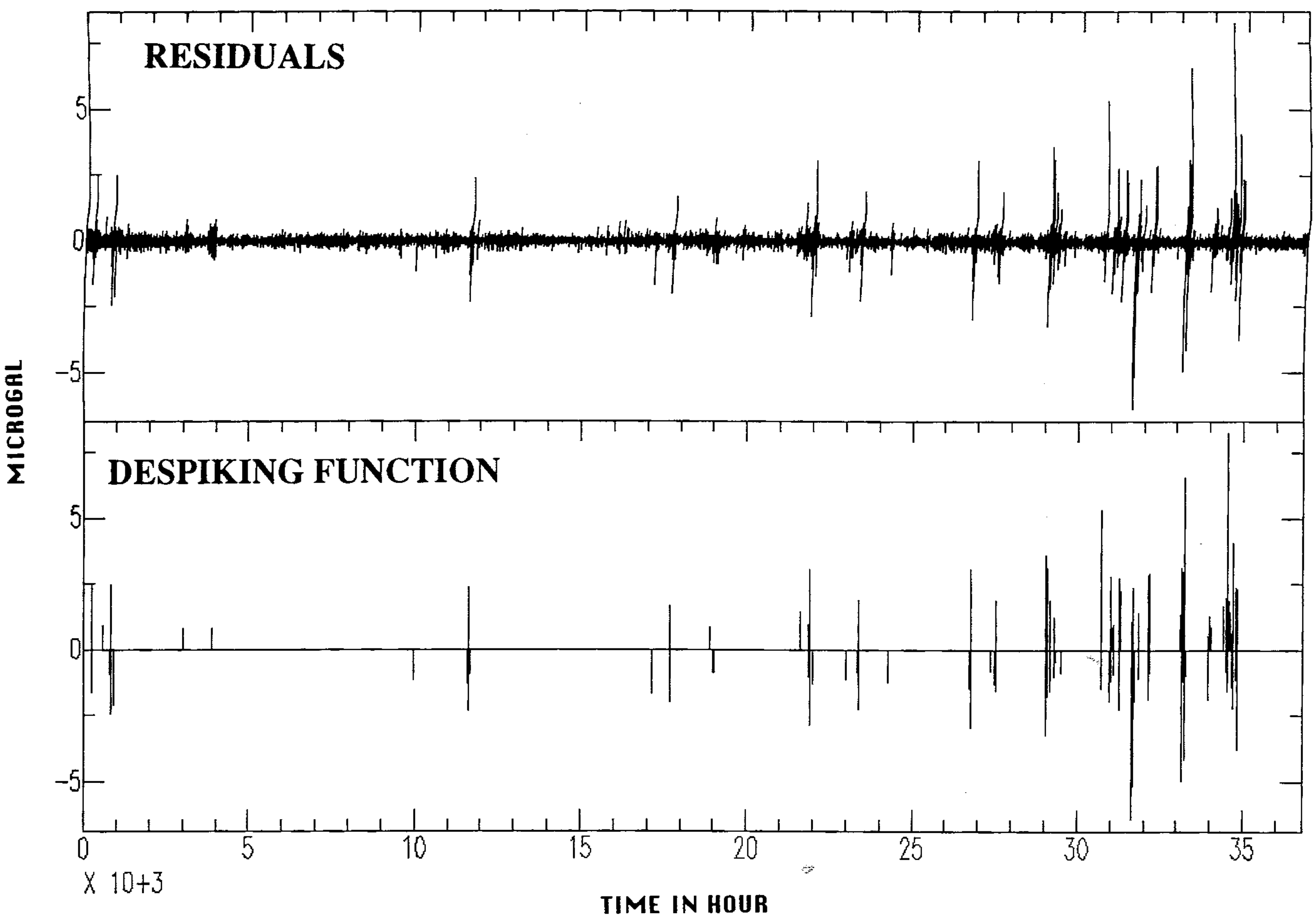


**Figure 1.** Despiking procedure on gravity residuals. The top part shows the gravity residuals after a least-squares fit of theoretical and local atmospheric pressure with the high-pass filtered gravity data. The length of the signal is 36 792 hourly values (4.2 years). The bottom part shows the despiking function that is obtained when choosing all spikes above three standard deviations.

Notice the change in the amplitude scale, the second residual signal now fluctuating within a $\pm 0.8\,\mu$gal range. Fig. 3 is the normalized amplitude spectrum of this signal (a pure harmonic signal of unit amplitude in time would appear as a unit spike in the amplitude spectrum). There is still some residual energy of tidal origin in the 1–4 cycle per day frequency bands. The left part of the spectrum is simply the result of the high-pass filtering procedure (cut-off frequency of 1/3 cycle per day) which was applied to the raw gravity before despiking. The despiking procedure is difficult to apply if the signal contains low-frequency components, which cannot be well accounted for in the least-squares fit (e.g. long-period atmospheric loading, unmodelled drift or non-tidal contributions). For this reason, the gravimetric signal is first of all high-passed with a finite impulse response (FIR) filter shown on Fig. 4. The filter in the time domain is more than two weeks long (361 hourly coefficients). The cut-off period is three days (0.0139 cycle per hr in frequency). The residual ripples in the tidal frequency bands (1 cycle per day and higher frequencies) is less than $10^{-5}$ in amplitude. Hence the effect of the high-pass filter on the delta factors is negligible. Most of the instrumental drift, as well as other long-term contributions of atmospheric origin, are removed by this filter.

Table 1 gives the $\bar{\delta}$ factors (modulus 'delta' and phase 'kappa') with their RMS uncertainties of the main tidal groups we introduced in the least-squares fit. We used 21 groups in the diurnal band, 16 in the semi-diurnal and one in the 10-diurnal one. A total of 36 792 hourly gravity values (4.2 years) from 11 October 1987 to 23 December 1991 were analysed. Fig. 5 shows the modulus of the $\bar{\delta}$ factors in the semi-diurnal band. The tidal waves determined with small uncertainties ($2N_2$, $\mu_2$, $N_2$, $\nu_2$, $M_2$, $L_2$, $T_2$, $S_2$, $K_2$) do not show a constant amplitude factor but rather a kind of bell pattern (the peak value being for $L_2$ and the tails for $2N_2$ and $K_2$). We do not have any explanation of this phenomenon for the moment; probably, a better evaluation of atmospheric or ocean effects is necessary to understand this problem.

## DIURNAL TIDAL WAVES AND NDFW COMPLEX EIGENFREQUENCY DETERMINATION

The possibility of resonance of the liquid core of the Earth has been known since the studies of Hough (1895) and Poincaré (1910). The elastic properties of the core were only

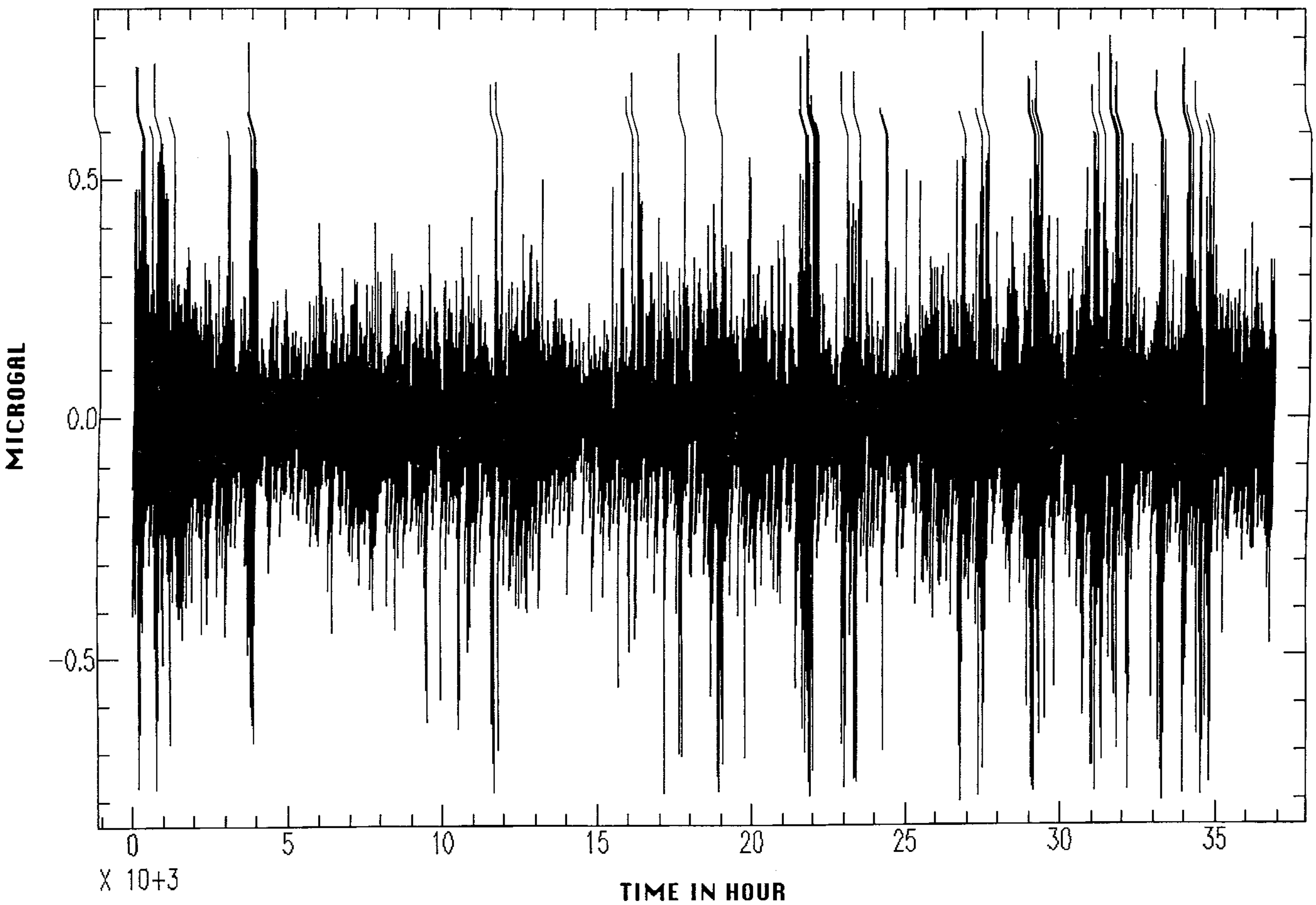


**Figure 2.** Residual gravity signal after the despiking procedure.

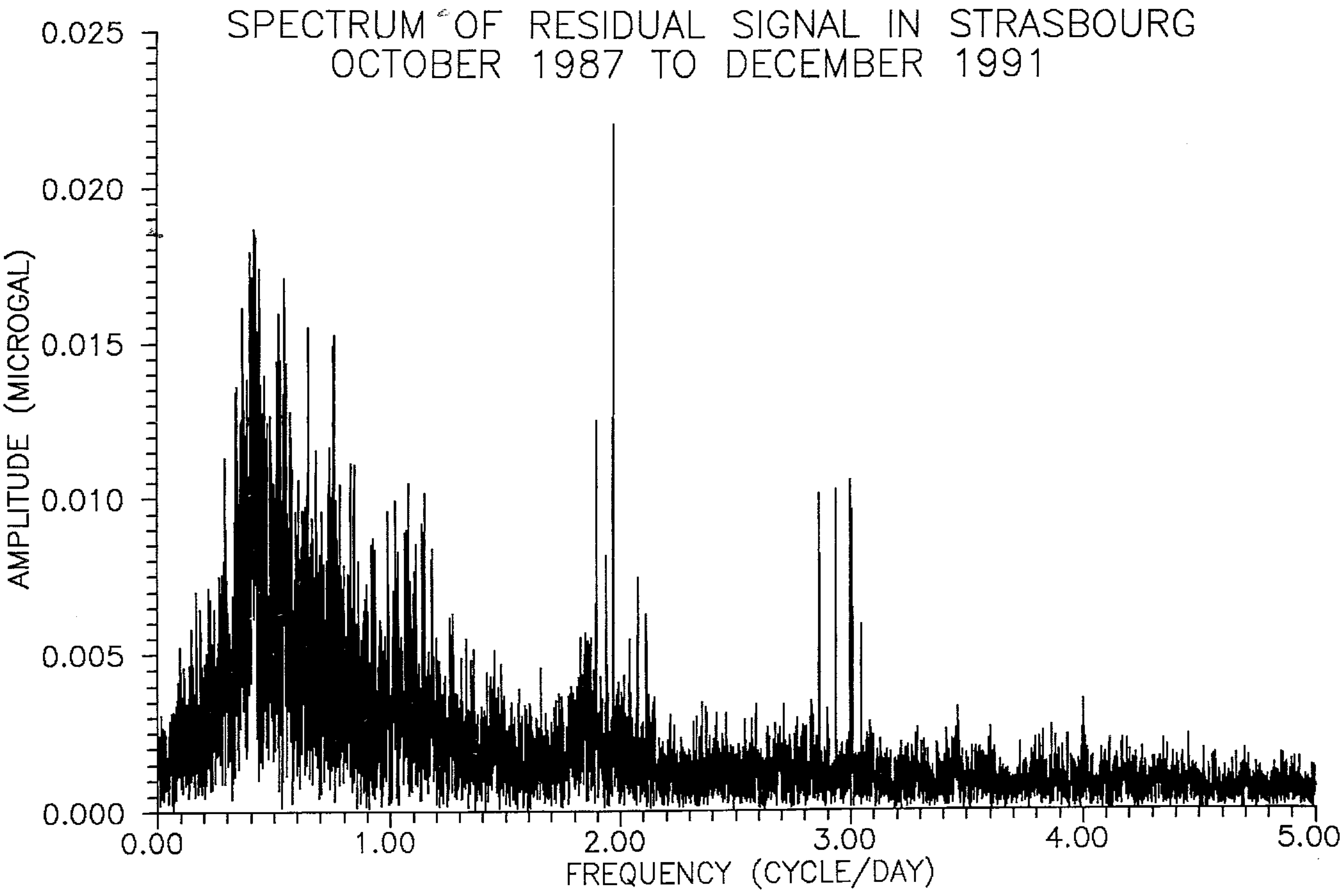


**Figure 3.** Normalized amplitude spectrum of the gravity residual signal shown on Fig. 2.

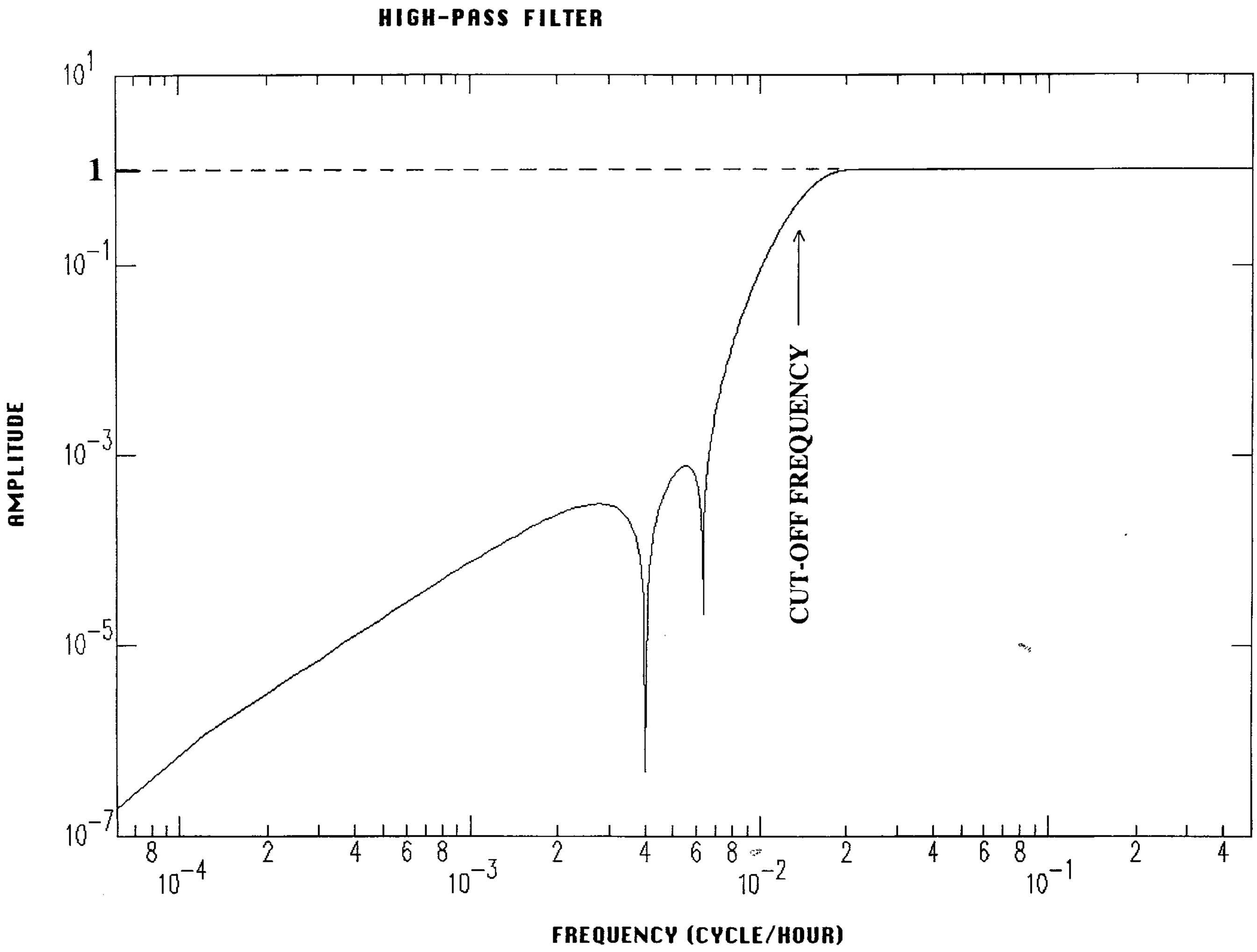


**Figure 4.** Amplitude spectral response of the high-pass filter applied to the gravity data before tidal analysis. The cut-off period is three days (0.0139 cycle $hr^{-1}$ in frequency).

introduced later by Jeffreys & Vicente (1957), and Molodensky (1961). These studies have been extended by several authors; especially important are the studies by Sasao, Okubo & Saito (1980) and Wahr (1981). In the Earth's rotating reference frame, the existence of the fluid outer core causes a retrograde motion of the instantaneous axis of rotation of the Earth with respect to its figure axis, with a period close to 1 sidereal day and is called NDFW. The theoretically predicted eigenfrequency of this mode is $-\Omega(1+\varepsilon)$, where $\Omega$ is the sidereal frequency and $\varepsilon \ll 1$ depending on the theory. Typically, $\varepsilon$ ranges from 1/460 to 1/475 (see e.g. Neuberg, Hinderer & Zürn 1990). In the inertial reference frame, the associated spatial nutation has a long period (frequency $\Omega\varepsilon$) and is called the FCN.

Attempts to detect the resonance effect related to the NDFW on tidal deformation began after the International Geophysical Year (1957). The first significant result was obtained by Lecolazet & Melchior (1975) in gravimetry and Levine (1978) in strain measurements. More recently, the increasing accuracy of tidal measurements provided by superconducting gravimeters prompted new investigations (Warburton & Goodkind 1978; Goodkind 1983; Zürn, Rydelek & Richter 1986; Neuberg, Hinderer & Zürn 1987; Zürn & Rydelek 1991), where the eigenperiod and damping of the FCN could be retrieved. VLBI measurements of the same resonance effect affecting lunisolar spatial nutations can also provide similar information on the FCN parameters (Gwinn, Herring & Shapiro, 1986; Gwinn & Shapiro, Herring 1986). Details on the numerical procedure used to retrieve the NDFW parameters from gravity data can be found in Neuberg *et al.* (1987). They have developed a 'stacking' method based on the Marquardt algorithm which allows the inclusion of simultaneous measurements from different stations to infer the parameters of a damped harmonic oscillator.

Instead of their linearized least-squares estimation, we use here a simpler approach and try to find an analytical solution to the problem. The complex gravimetric factor $\tilde{\delta}$ of a tidal wave of frequency $\sigma$ can be written (Hinderer 1986; Neuberg *et al.* 1987):

$$\tilde{\delta}=\left(1+h-\frac{3k}{2}\right)+\frac{A\left(h_1-\frac{3k_1}{2}\right)\left(\alpha-\frac{q_0h^c}{2}\right)\Omega}{A^m(\sigma-\tilde{\sigma}_{nd})}, \quad (3)$$

where $h$, $k$, $h_1$, $k_1$ and $h^c$ are Love numbers appearing

**Table 1.** Tidal results from a 4.2 yr data set recorded with the Strasbourg superconducting gravimeter. The gravimetric factor (delta) and phase (kappa) are derived from a tidal least-squares fit using despiked and high-passed gravity signal and are given with their RMS uncertainty (phases are given in degrees).

| TIDE | $\delta$ | RMS | $\kappa$ | RMS |
|---|---|---|---|---|
| $\sigma Q_1$ | 1.1521 | 0.0110 | -0.54 | 0.55 |
| $2Q_1$ | 1.1506 | 0.0035 | -0.65 | 0.17 |
| $\sigma_1$ | 1.1490 | 0.0030 | -0.30 | 0.15 |
| $Q_1$ | 1.1449 | 0.0005 | -0.41 | 0.05 |
| $\rho_1$ | 1.1469 | 0.0025 | -0.46 | 0.13 |
| $O_1$ | 1.1471 | 0.0001 | -0.10 | 0.00 |
| $\tau_1$ | 1.1654 | 0.0094 | 0.25 | 0.46 |
| $MM_1$ | 1.1434 | 0.0119 | 0.22 | 0.60 |
| $M_1$ | 1.1508 | 0.0011 | -0.21 | 0.05 |
| $\chi_1$ | 1.1455 | 0.0060 | -0.08 | 0.30 |
| $\pi_1$ | 1.1371 | 0.0039 | 0.34 | 0.19 |
| $P_1$ | 1.1485 | 0.0002 | 0.05 | 0.01 |
| $S_1$ | 1.1933 | 0.0143 | 0.53 | 0.69 |
| $K_1$ | 1.1354 | 0.0001 | 0.10 | 0.00 |
| $\psi_1$ | 1.2589 | 0.0093 | 0.40 | 0.42 |
| $\phi_1$ | 1.1698 | 0.0054 | 0.47 | 0.26 |
| $\theta_1$ | 1.1546 | 0.0062 | -0.03 | 0.31 |
| $J_1$ | 1.1588 | 0.0011 | -0.10 | 0.05 |
| $SO_1$ | 1.1596 | 0.0068 | 0.15 | 0.34 |
| $OO_1$ | 1.1534 | 0.0015 | -0.13 | 0.07 |
| $V_1$ | 1.1735 | 0.0074 | -0.80 | 0.36 |
| $\epsilon_2$ | 1.1521 | 0.0117 | 2.31 | 0.58 |
| $2N_2$ | 1.1540 | 0.0036 | 2.55 | 0.18 |
| $\mu_2$ | 1.1545 | 0.0033 | 2.24 | 0.16 |
| $N_2$ | 1.1745 | 0.0005 | 2.26 | 0.03 |
| $\nu_2$ | 1.1721 | 0.0028 | 2.34 | 0.14 |
| $\alpha_2$ | 1.2028 | 0.0219 | 1.06 | 1.04 |
| $M_2$ | 1.1849 | 0.0001 | 1.81 | 0.00 |
| $\beta_2$ | 1.2584 | 0.0327 | 3.26 | 1.49 |
| $\lambda_2$ | 1.1913 | 0.0141 | 1.56 | 0.68 |
| $L_2$ | 1.2111 | 0.0045 | 0.55 | 0.21 |
| $T_2$ | 1.1892 | 0.0037 | 0.19 | 0.18 |
| $S_2$ | 1.1881 | 0.0002 | 0.30 | 0.01 |
| $R_2$ | 1.1905 | 0.0211 | -0.01 | 1.02 |
| $K_2$ | 1.1884 | 0.0006 | 0.55 | 0.03 |
| $\xi_2$ | 1.1684 | 0.0557 | -1.45 | 2.73 |
| $\eta_2$ | 1.1850 | 0.0088 | -0.89 | 0.42 |
| $M_3$ | 1.0661 | 0.0035 | -0.04 | 0.19 |

because of the elastic-gravitational deformation (Hinderer & Legros 1989). $A$ and $A^m$ are the equatorial moments of inertia of the Earth and mantle, respectively, $\alpha$ is the dynamical flattening and $q_0$ the geodynamical constant expressing the ratio of the centrifugal force to the mean gravity at the Earth's surface, $\tilde{\sigma}_{nd}$ is the complex eigenfrequency to be determined. In the case of a purely elastic model, the Love numbers remain real quantities, but, if some anelasticity is considered, they become complex numbers (e.g. Wahr & Bergen 1986). Considering only three different tidal waves, eq. (3) can be rewritten ($j = 1, 2, 3$):

$$\tilde{\delta}_j = \tilde{\delta}_o + \frac{\tilde{a}}{\sigma_j - \tilde{\sigma}_{nd}}, \tag{4}$$

where $\tilde{\delta}_0$ and the strength $\tilde{a}$ are assumed to be frequency-independent (see e.g. Hinderer, Zürn & Legros 1991b). We obtain a system of three complex equations with three complex unknowns $\tilde{\delta}_0$, $\tilde{a}$ and $\tilde{\sigma}_{nd}$.

This system can be analytically solved and the solution is:

$$\tilde{\delta}_0 = \frac{\sigma_1\tilde{\delta}_1(\tilde{\delta}_3 - \tilde{\delta}_2) + \sigma_2\tilde{\delta}_2(\tilde{\delta}_1 - \tilde{\delta}_3) + \sigma_3\tilde{\delta}_3(\tilde{\delta}_2 - \tilde{\delta}_1)}{\sigma_1(\tilde{\delta}_3 - \tilde{\delta}_2) + \sigma_2(\tilde{\delta}_1 - \tilde{\delta}_3) + \sigma_3(\tilde{\delta}_2 - \tilde{\delta}_1)},$$

$$\tilde{\sigma}_{nd} = \frac{\sigma_1\tilde{\delta}_1(\sigma_2 - \sigma_3) + \sigma_2\tilde{\delta}_2(\sigma_3 - \sigma_1) + \sigma_3\tilde{\delta}_2(\sigma_1 - \sigma_2)}{\sigma_1(\tilde{\delta}_3 - \tilde{\delta}_2) + \sigma_2(\tilde{\delta}_1 - \tilde{\delta}_3) + \sigma_3(\tilde{\delta}_2 - \tilde{\delta}_1)}, \tag{5}$$

$$\tilde{a} = -\frac{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3)(\sigma_3 - \sigma_1)(\tilde{\delta}_1 - \tilde{\delta}_2)(\tilde{\delta}_2 - \tilde{\delta}_3)(\tilde{\delta}_3 - \tilde{\delta}_1)}{[\sigma_1(\tilde{\delta}_3 - \tilde{\delta}_2) + \sigma_2(\tilde{\delta}_2 - \tilde{\delta}_3) + \sigma_3(\tilde{\delta}_2 - \tilde{\delta}_1)]^2}$$

where $\sigma_i$ ($i = 1, 2, 3$) are the frequencies of the three selected waves. The error estimates on the unknowns can be analytically computed from the errors on the tidal parameters ($\delta_i$, $\kappa_i$) using partial derivatives. We have tested an alternative method in which only two waves are taken into account, but assuming $\tilde{\delta}_0$ is known: it would be the mean $\tilde{\delta}$ in the absence of the resonance. The unknown $\tilde{\delta}_0$ being eliminated, the system is then linear. However, since the diurnal domain is fully affected by the resonance, choosing this reference factor is not obvious. Actually, we suggest the use of at least three waves. When using more than three waves, the system becomes overdetermined and must then be solved by using a least-squares method as in Neuberg *et al.* (1987).

Which tidal waves to choose? Choosing the three waves from the 21 waves in the diurnal tidal frequency band seems quite arbitrary. To help in the decision, the following criteria were used: (1) the best waves are those which are highly affected by the resonance; (2) the waves must be determined with a high accuracy, especially if their frequency is far from the resonance frequency; (3) those waves open to atmospheric disturbances must be avoided; (4) the ocean load correction must be well known or negligible.

Taking into account these criteria, we suggest the use of $O_1$, $K_1$ and $\psi_1$. Indeed, $O_1$ is very well determined, while far from the resonance. $K_1$ is also known with relative accuracy and is close to the NDFW frequency. $\psi_1$ is not very accurate because of its small amplitude, but is the closest wave to the resonance. $S_1$ has to be avoided because of the existence of a thermal planetary tide of the same frequency in the atmosphere (Haurwitz & Cowley 1973); we did not use $P_1$ as it is also affected by the thermal tide $S_1$ through annual modulation (e.g. Legros & Hinderer 1991). Of course, any set of three tidal waves will give a solution. We did some tests with other choices of three waves indicating a rather good stability of the eigenperiod but also noticeable differences in the damping value. Moreover the

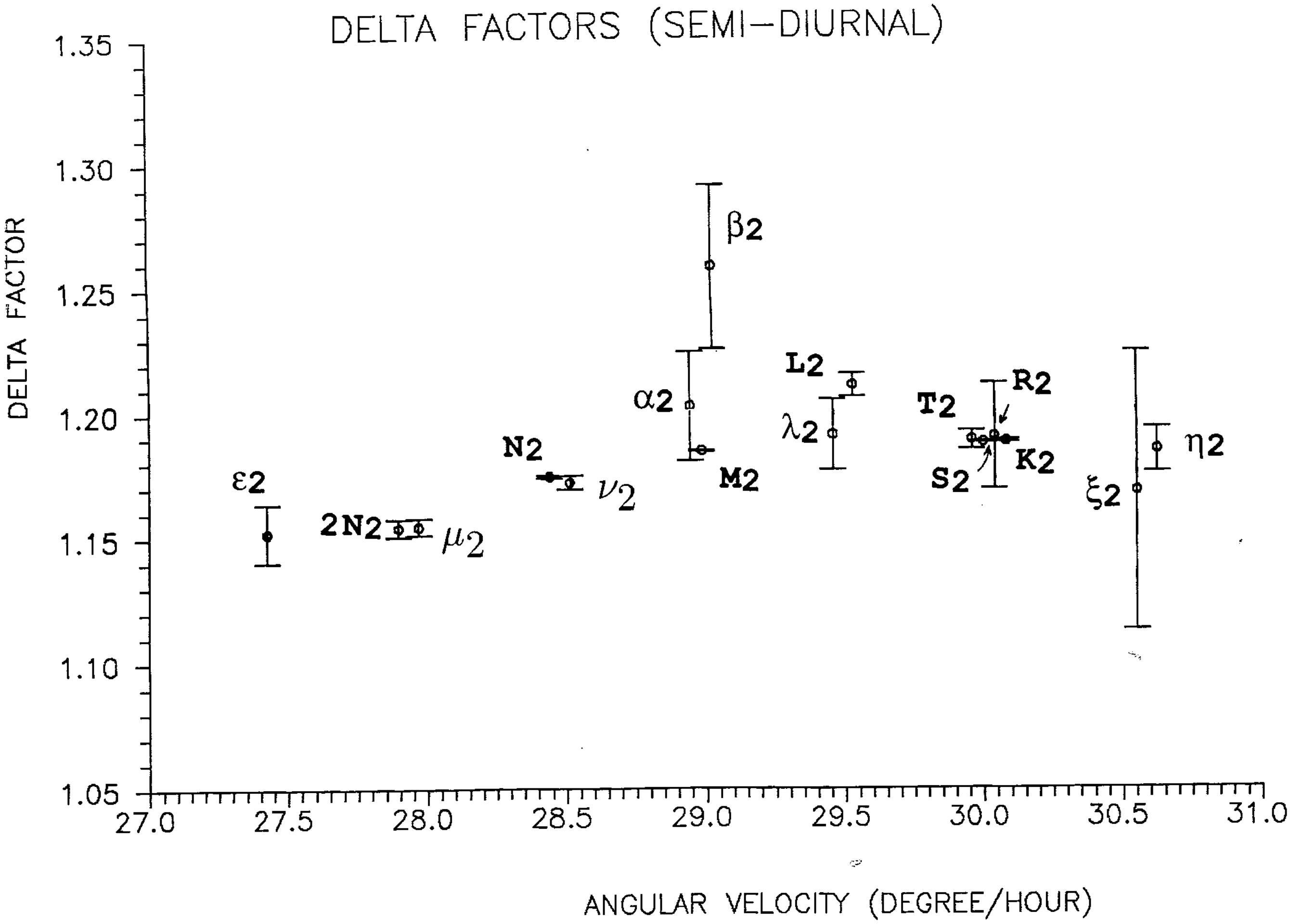


**Figure 5.** Gravimetric amplitude factors in the semi-diurnal tidal band. The uncertainties are the RMS errors given by the tidal least-squares analysis.

uncertainties were always larger than the ones obtained when selecting $O_1$, $K_1$ and $\psi_1$; in our opinion, this reflects the effect of the above-mentioned disturbances on the other waves in retrieving the NDFW parameters, or the fact that the waves are too far away in frequency from the resonance.

The amplitudes of these waves are affected by ocean load phenomena and have to be corrected before the analytical method can be applied. The ocean load corrections for the waves $O_1$ and $K_1$ were computed with a program allowing regional ocean-tide modelling (Scherneck 1991). The model is the Schwiderski (1980) global ocean model everywhere except in the North Atlantic, where Schwiderski's model is replaced by the Flather (1976) model (see also the discussion on $\tilde{\delta}(O_1)$ in Hinderer *et al.* 1991c). The correction for the small wave $\psi_1$ is of course not known but is much smaller than the uncertainty on the tidal amplitude itself (e.g. Zürn *et al.* 1986).

## RESULTS AND DISCUSSION

Figure 6 shows the amplitudes of the gravimetric factor of diurnal tidal waves and the NDFW resonance curve. Fig. 7 is a magnification of Fig. 6 in the vicinity of the resonance. The effect of the ocean load corrections is shown on both figures. Since the solution for the three chosen resonant waves ($O_1$, $K_1$, $\psi_1$) is analytical, the resonance curve fits exactly the gravimetric factors for these waves. In the absence of damping, the curve would be a simple hyperbola. The dissipation is responsible for the slightly different form of the curve. The quality factor of the eigenmode can be written $Q = \sigma_r/2\sigma_i$ (e.g. Neuberg *et al.* 1987), where $\sigma_i$ and $\sigma_r$ are the imaginary and real part of the complex eigenfrequency $\tilde{\sigma}_{\rm nd}$. The main results of the analytical method are given in Table 2 where we also report results from other studies. The results for the Strasbourg spring gravimeter are from a one-year record of a Lacoste–Romberg gravimeter (LCR) with electrostatic feedback (Abours & Lecolazet 1979). The results for the Brussels station come from a three-year data set of the Belgium SG (Ducarme *et al.* 1986). The record of the Bad Homburg SG was also three years long (Richter & Zürn 1986). The higher quality of the superconducting gravimeter for an accurate determination of tides and retrieval of NDFW resonance parameters is apparent from Table 2, as compared with results for the Lacoste–Romberg gravimeter at the same station. The uncertainty on the period and damping are greatly reduced and, even more important, the mean values ($T_{\rm nd} = 430.7$ days, $Q = 2080$) are in much better agreement with the two other SG results (Brussels and Bad Homburg), as well as with VLBI results. There are some other recent

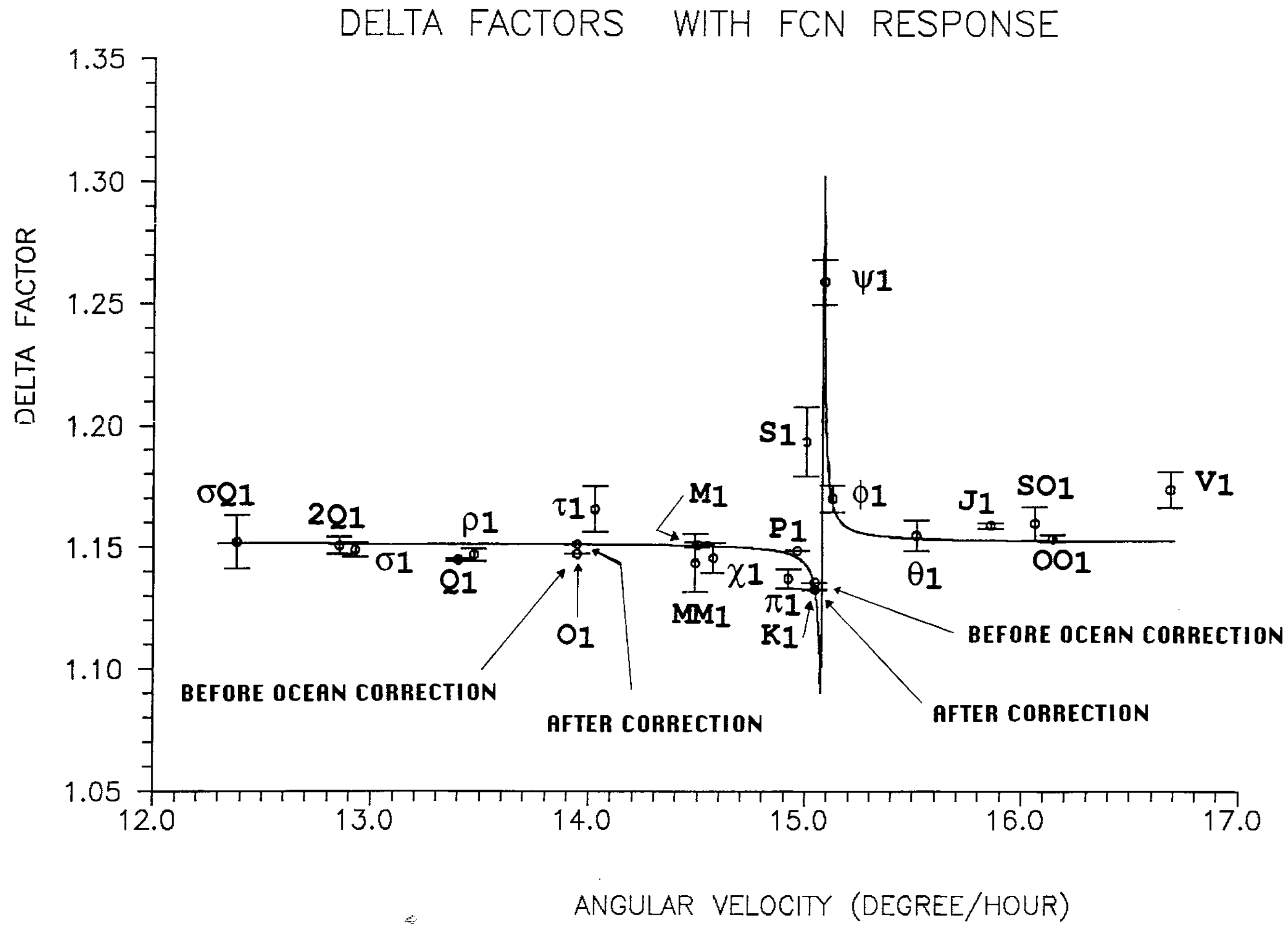


**Figure 6.** Gravimetric amplitude factors in the diurnal tidal band. The uncertainties are the rms errors given by the tidal least-squares analysis. The effect of the ocean load correction is shown on $O_1$ and $K_1$. The solid line is the NDFW resonance curve derived from our analytical method applied to the waves $O_1$, $K_1$ and $\psi_1$.

studies investigating the nearly diurnal resonance effect in tidal gravity data and strain data (Hsu & Hua 1991; Sato 1991; Zürn & Rydelek 1991; Rydelek *et al.* 1991) which we did not report since we focus here on results obtained with superconducting gravimeters.

Our analytical method provides a confidence interval even smaller than in previous studies on superconducting gravity data and comparable with the small uncertainty achieved by VLBI techniques. This again confirms the reality of the discrepancy between theory and observation for the NDFW period which is shown by Table 2. The most easy explanation proposed till now for the period reduction is an increase of about 5 per cent in the CMB dynamical flattening (Gwinn *et al.* 1986; Wahr & de Vries 1989). Notice that mantle anelasticity would even increase the value of this flattening because it lengthens the theoretical eigenperiod by a few days (Wahr & Bergen 1986).

The damping of the NDFW remains more problematic. Basically, our value shows that the $Q$ factors obtained from superconducting gravity data are always much smaller than the ones inferred from VLBI, even if the uncertainty in the latter is larger. It is known that mantle anelasticity is able to decrease $Q$ factors from infinity to a finite value close to 80 000 (Wahr & Bergen 1986), but, at least, one order of magnitude is still missing. Other mechanisms dealing with the coupling between the fluid core and the solid mantle have also been investigated without success (Neuberg *et al.* 1990). It appears that, if one attributes the damping of the NDFW to viscous core–mantle coupling, the required values of core viscosity would be larger by many orders of magnitude (Neuberg *et al.* 1990; Lumb & Aldridge 1991) than the laboratory experimental values (Poirier 1988). We are therefore left with the possibility of systematic errors affecting gravimetric, VLBI or both observation methods. In the superconducting gravimeter, one problem could be the calibration factor of the instrument itself, or a time dependence of this factor. The Strasbourg instrument was first calibrated against absolute gravity measurements with a precision of the order of 1 per cent (Hinderer *et al.* 1991c). Later on, we could successfully check this value with the calibration factor inferred from a parallel registration of two Lacoste–Romberg gravity meters which were operating for several months at our station. On the other hand, it is very hard to assess any evolution in time of the calibration factor. Anyway, possible errors due to inaccurate calibration play very little role in the fit, as shown recently by Zürn & Rydelek (1991). More details on error estimates in the NDFW parameters retrieval can also be found in this last study.

The next source of systematic error may be in the ocean

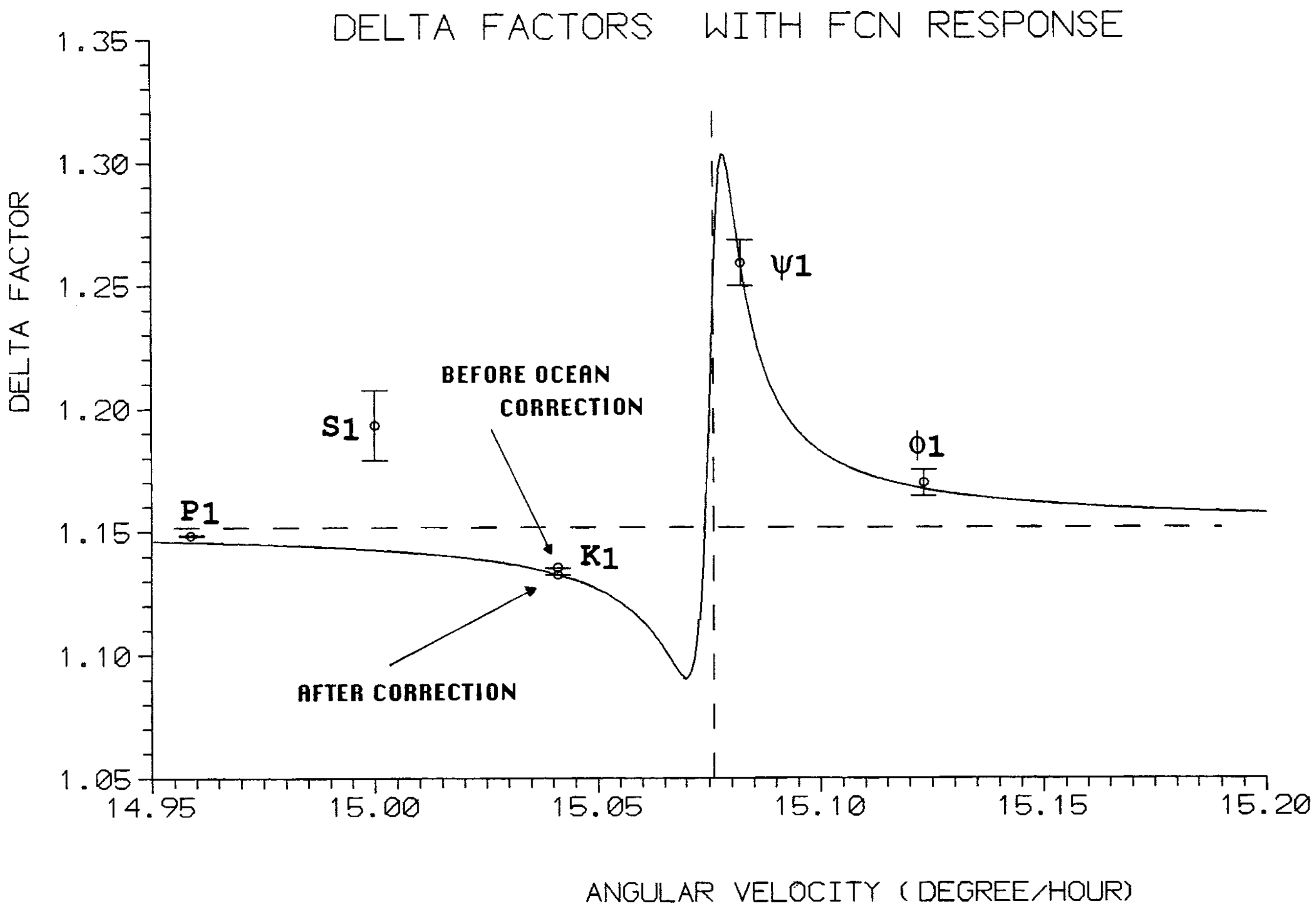


**Figure 7.** Gravimetric amplitude factors near the NDFW frequency. This figure is a magnified version of Fig. 6 for tidal waves of frequency close to the diurnal eigenfrequency.

load corrections. This has already been investigated to some extent by Neuberg *et al.* (1987) with a Monte Carlo approach. Especially interesting was the induced spread in the values of the NDFW period and damping when considering ocean load corrections with a random error of 40 per cent of the nominal computed correction; the dispersion was indeed larger in $Q$ than in period, but the extreme $Q$ values hardly exceeded 4000 (the mean value being close to 2800). Of course, any ocean load error for the tides used to retrieve the FCN parameters would not only act on gravity but also on the associated spatial nutations (see e.g. Legros & Hinderer 1991). There is, however, a difference in the sense that the gravity values reported here come from stations all located in Central Europe, while the VLBI results are inferred from a world-wide network. So, a systematic bias caused by ocean load effects in Europe cannot be totally excluded until other superconducting gravimeter records from stations outside Europe have been analysed.

Like ocean loading, atmospheric loading is another possible source of error. The same remarks made above for the ocean loading process apply, except that the additional atmospheric refraction correction is only affecting nutations estimates. Once again, the barometric pressure corrections for the European gravity stations, which are only corrections dependent on local atmospheric admittances, might be a further source of discrepancy with VLBI nutation estimates.

## CONCLUSIONS

The analysis of more than four years of gravity data from the Strasbourg superconducting gravimeter has provided an accurate determination of the tidal waves affected by the NDFW resonance effect. We have shown the primary importance of the 'despiking' procedure applied to tidal gravity residuals in order to achieve a high precision in the determination of the Earth's elastogravitational response to the tidal forcing. A new simple algorithm was implemented to extract analytically from the amplitude and phase of three tidal waves the period and damping of the NDFW. These values could be much better determined than in a previous study using a record from a Lacoste–Romberg spring meter at the same station. The eigenperiod we found is in close agreement with studies involving other SG data in Europe (Brussels and Bad Homburg), as well as with VLBI results. This is an additional confirmation of the reality of the discrepancy between the observed and theoretical NDFW period. Moreover, our study also confirms that the value of the damping inferred from superconducting gravimetry is always larger than the one deduced from VLBI. However, our station being located in the same region as the two other SG, we cannot dismiss any regional bias which would be the consequence of specific ocean or atmospheric loading in Central Europe. It clearly appears that it will be necessary in future to obtain a stack of different superconducting

**Table 2.** Results for the NDFW period and damping. SG stands for superconducting gravimeter, LCR for Lacoste–Romberg spring meter and VLBI for very long baseline interferometry.

| | Period (sidereal day) | Q factor |
|---|---|---|
| Strasbourg SG | 430.7 ± 1.0 | 2080 ± 440 |
| (this study) | | |
| Strasbourg LCR | 413.3 ± 15.2 | 8547 ± 12930 |
| (Zürn and Rydelek 1991) | | |
| Bad Homburg SG | 431.2 ± 3.0 | 3125 ± 322 |
| (Richter and Zürn 1986) | | |
| Brussels SG | 439.4 ± 18.4 | 1876 ± 668 |
| (Zürn and Rydelek 1991) | | |
| Brussels + Bad Homburg SG | 431.0 ± 6.0 | 2781 ± 543 |
| (Neuberg et al. 1987) | | |
| VLBI | 434.6 ± 0.6 | 21740 ± 10397 |
| (Herring et al. 1986) | | |
| Elastic theory | 466.9 | ∞ |
| (Sasao et al. 1980) | | |
| Anelastic theory | 473.8 | 78125 |
| (Wahr and Bergen 1986) | | |

gravimeter records over a world-wide distribution including instruments in operations in China, Japan, the USA and Canada.

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