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Multifractals and statistical prediction of precipitations and floods

Abstract

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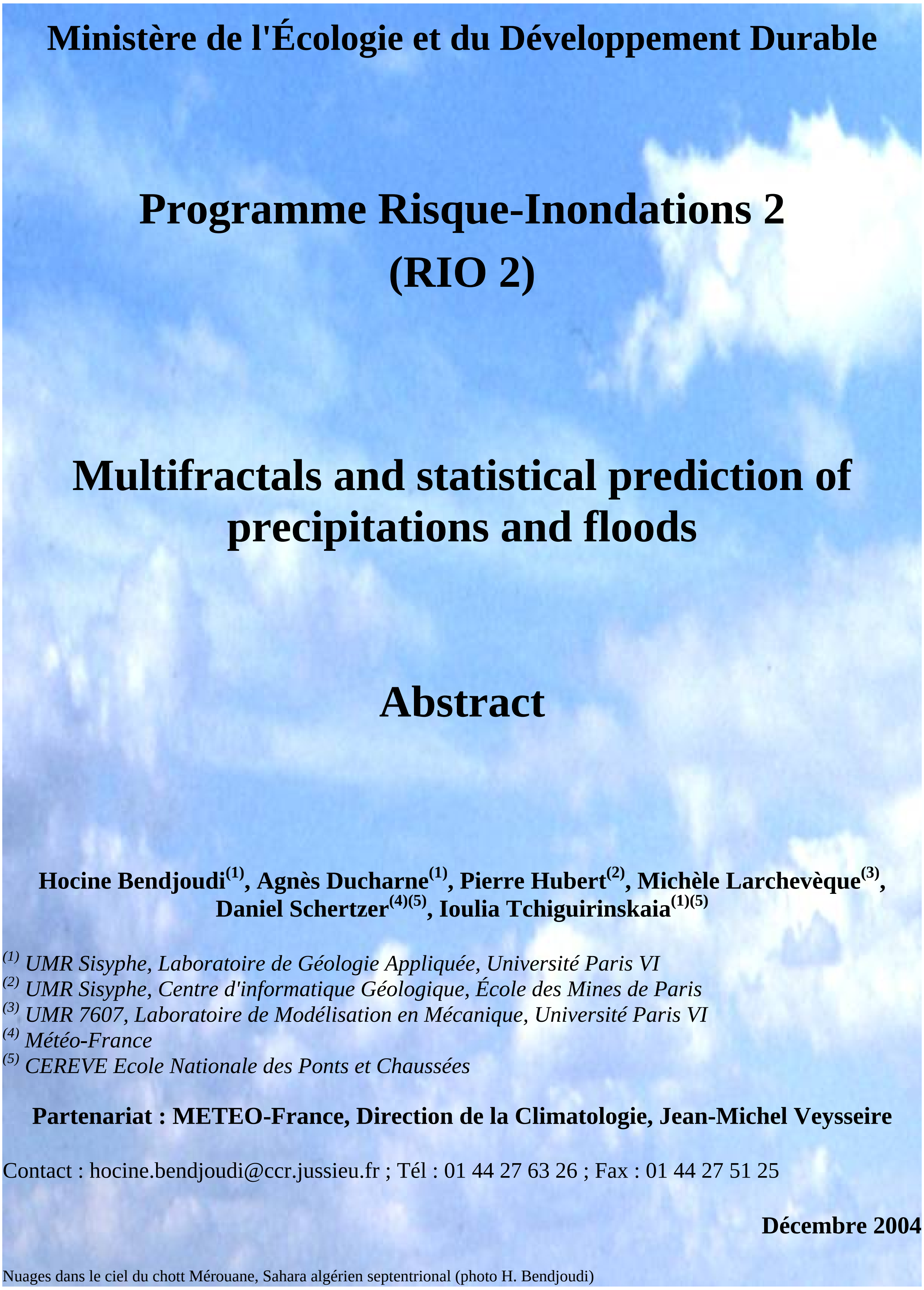

Nuages dans le ciel du chott Mérouane, Sahara algérien septentrional (photo H. Bendjoudi)

Extreme events are of prime importance in hydrology with their determining societal impact. They represent key variables not only for the forecast of the risks and engineering designs (dams, bridges, spillways) but also for the management of water resources and land uses.

The statistical behaviour of extremes is also appealing at the theoretical level because at least two types of "universal" theoretical laws were proposed up to now: exponential or algebraic. However the question is not only of theoretical interest: the practical consequences of the choice of the theoretical law of extremes are huge, because algebraic laws decrease much more slowly than exponential laws that are usually used for the determination of events of given recurrence, which would then be considerably underestimated.

Our ambition in this study was to approach the predetermination of extremes in hydrology by taking hold on the physics of the involved phenomena, instead of limiting ourselves to either a purely statistical approach, or a so-called "physically based" approach, taking into account only a limited range of scales. On the contrary, our approach and our analyses took systematically into account the wide multiplicity of scales generating the extremes.

In this framework, our approach corresponds to consider that the system rainfall-discharge, especially their extremes, can be analyzed and modelled as system having a multifractal source (the precipitation) interacting with a multifractal environment (the basin) and producing a multifractal output (the discharge).

At first we recalled and illustrated that in a general manner the cascade phenomena, which have been often used to model the extreme variability of the rain on wide ranges of spatiotemporal scales, easily lead to power laws tails of the probability laws. This is particularly well verified for the precipitation, since numerous studies conclude on such a law with an exponent $q_D \approx 3$. We empirically verified this multifractal behaviour of the precipitation by using the base PRECIP of Météo-France. We have to expect a similar behaviour for the maxima of a series. More precisely, if the series would have only short correlations, the law of extremes would be a Fréchet law (often called Log-Gumbel or a "Generalized Extreme Value" (GEV) of type 2), with the same exponent q_D, whose opposite $\gamma = 1/q_D$, is the shape parameter of the GEV. We empirically showed that the Fréchet law is far better adapted than the Gumbel law to describe its extremes.

To extend this analysis to the discharges, we were interested in the rainfall-discharge relationships, by beginning to prepare the ground of an approach which does not limited to analyzing time series of discharges in the outlet of a basin or in its some generally very limited number of measurement points. On the contrary, we wanted to understand, to analyze and to represent both in space and in time the processes which take place in a basin or even in a hydrological zone, in particular the fluxes across the multiple involved space-time scales. This brought to us to consider the relations between several multifractals fields: discharges, specific discharges, drainage areas, precipitations and the measure network itself. This last field allows, with the help of the multifractal "theorem of intersection", to infer the multifractal properties of the other fields from their rarely available measurements. These properties determine in return the behaviour of these fields almost in every point of the space or the time, while we started with a very limited number of observations in particular with the respect of their space distribution.

The framework of this approach was developed and tested at first on the R-Artic Network data base (monthly discharges), then applied to daily discharges of 173 stations relatively well distributed by the Rhone-Mediterranée-Corse basin. We empirically pointed out two relatively different scaling regimes for the discharges. A "climatological" regime for periods larger than a year and a "meteorological" regime for shorter periods. For the precipitation, the transition occurs rather around some weeks, which corresponds to the “synoptic maximum”, i.e. the average time life of the planetary scale structures. This shift rather points out the basin integration of effect. Another manifestation of this effect is the fact that the climatological average multifractality (measured by the codimension C_1) is about ten times lower than that of the meteorological regime, while they have both the same multifractality degree (measured by the Levy index α) We discussed the interest and limits to represent this integration by a fractional integration of the precipitation. The interest is that we easily obtain from rain time series, synthetic discharges time series, which are visually very close to those observed. But we also pointed out that frequently occurs a difference for the extremes: because of the fractional integration, the synthetic time series has the same extreme behaviour as that of the precipitation, generally much stronger than that of the observed discharges.

The basin integration effect was therefore studied more in detail by analyzing over the whole range of available scales the relations between discharges, specific discharges and drainage areas. We put in evidence a stronger universality for the specific discharges than for the discharges, i.e. a strong independence of the former with respect to the structure of basins and more particularly with respect to the drainage areas distribution, and a given similarity between specific discharge and rain.

These properties lead to a factorization: the spatial variability of the discharges is the product of the basin response variability (characterized here by the drainage area distribution) by the rain variability. This explains at the same time the observed variability of q_D for the discharges, and that they are generally different from those of the precipitation, because modulated by the drainage area distribution.

A very practical result of this analysis was the finalization of a risk coefficient allowing to estimate the shape of the distribution of the extremes of the discharges from their average behaviour, which can be well estimated over relatively short periods. Indeed, this coefficient of risk is calculated from the multifractals exponents determining the average behaviour of the field. We tested this coefficient on the data of the Rhône-Méditerranée-Corse basin and so obtained from a single year of observation a risk map that is very coherent with the observations made over much longer periods.

A lot of work remains to be done to come up these results in the law and in the engineering. This is particularly necessary for a time when all the societies owe, and will more and more owe, to assume an objective increase of the hydrological risk owed in a greater vulnerability, which the past practices of the applied hydrology certainly often underestimated.

In this scope, an important prospect was opened, by the implementation of the working group “New theoretical and empirical features of hydrological extremes”. The objective of which is to overcome the current stage, where every institution and every institutional sector can define its own standards, for the calculation for example of the centennial flood. This stake is complex and particularly important: the challenge is to progress in a significant way in the field of extremes, by mobilizing the most recent approaches both in hydrology and in statistics.