

Daily and Monthly Groundwater Recharge and Baseflow Based on 30-year (1984-2013) Observations in Illinois

IGEM “Impact of Groundwater in Earth system Models”

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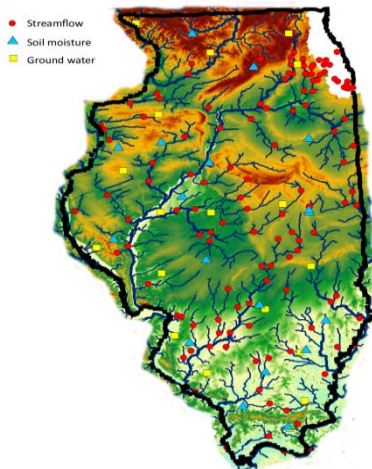
Outline

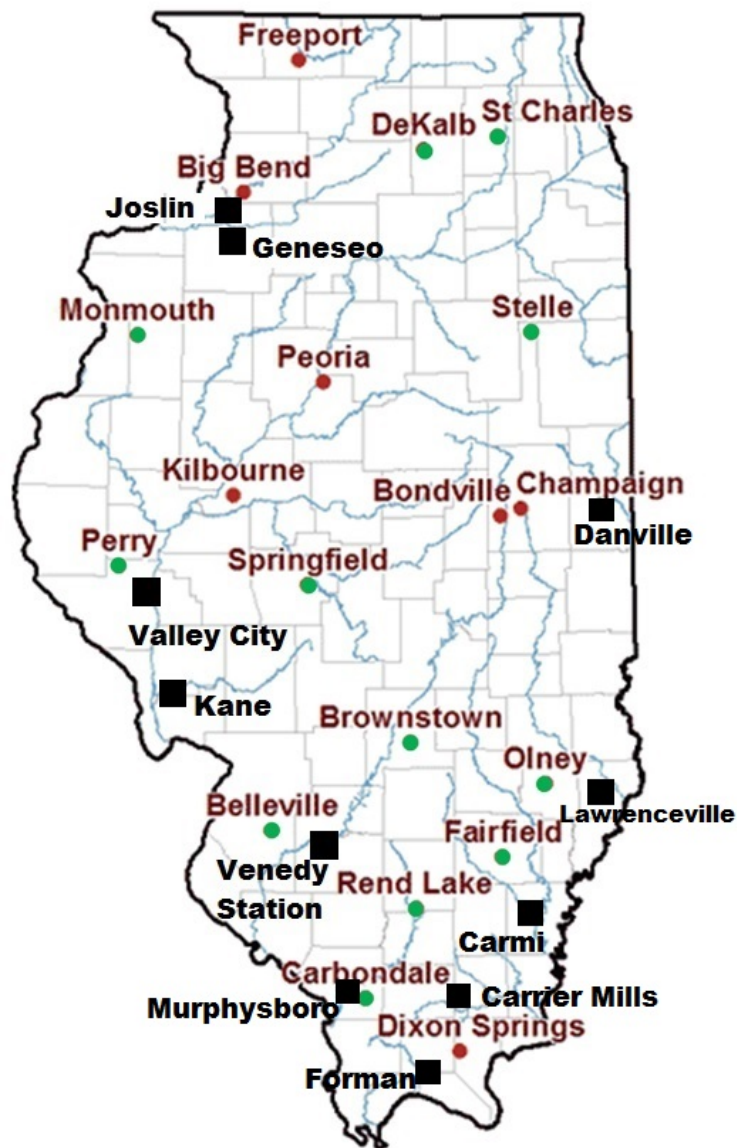
Part I

A 30-year (1984-2013) Monthly and Daily
**Groundwater Recharge and Baseflow
Estimates** in Illinois (regional-scale)

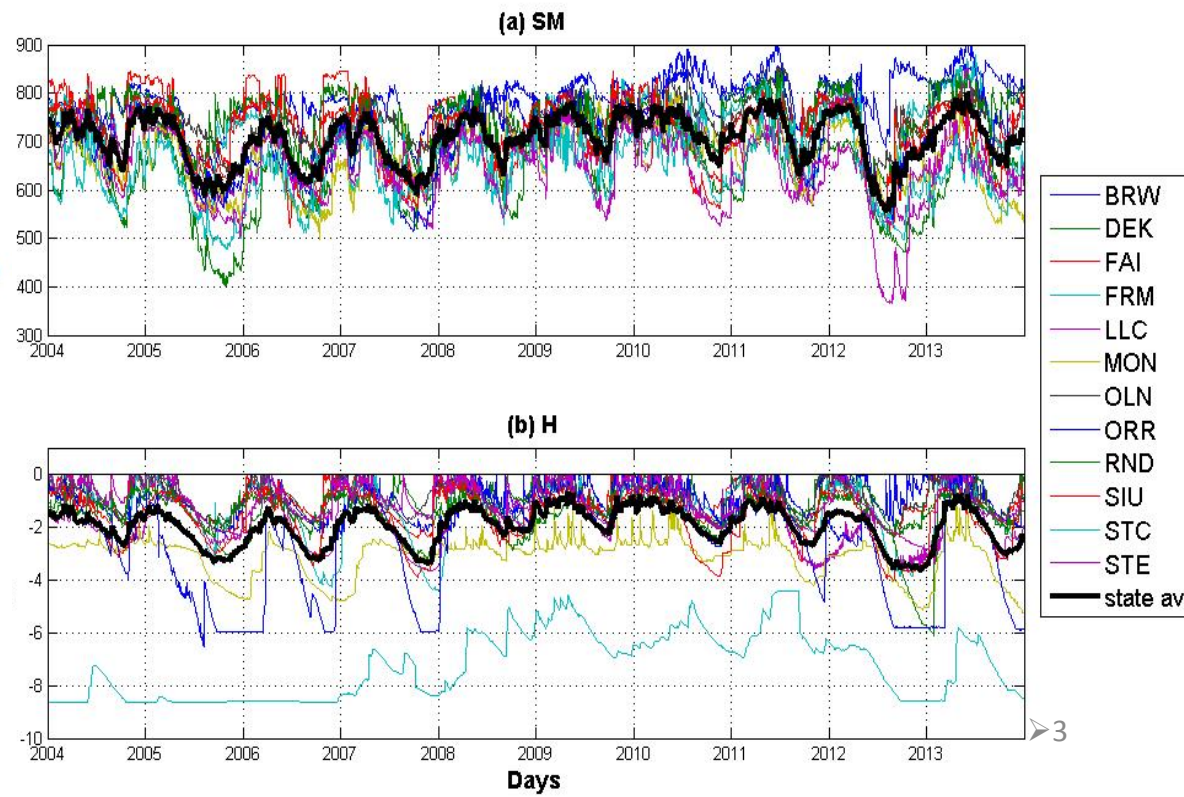
Part II

A 1997-2010 Daily Analysis of **Water Table
Depth** versus **Baseflow Recession** to
Characterize **Storage-Discharge** Relationship
in Illinois (local-scale)



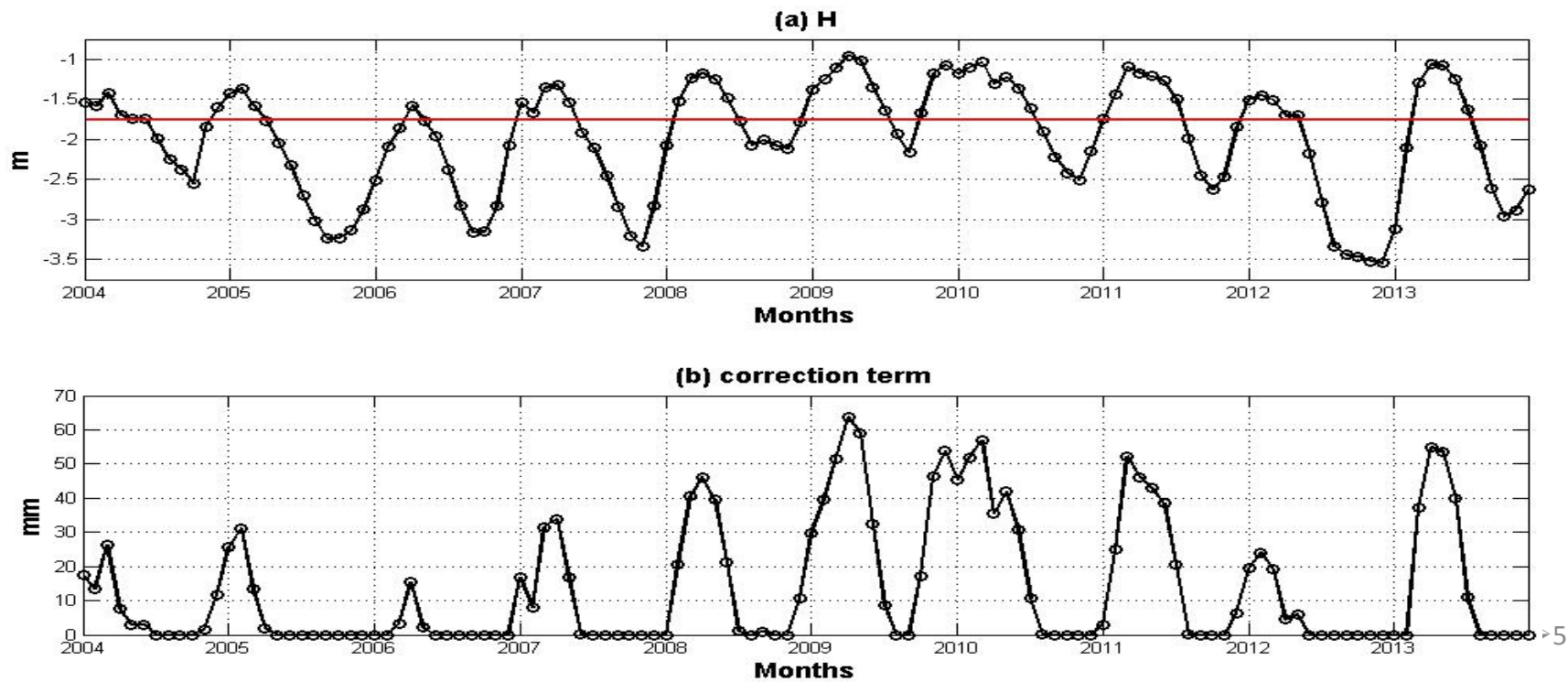
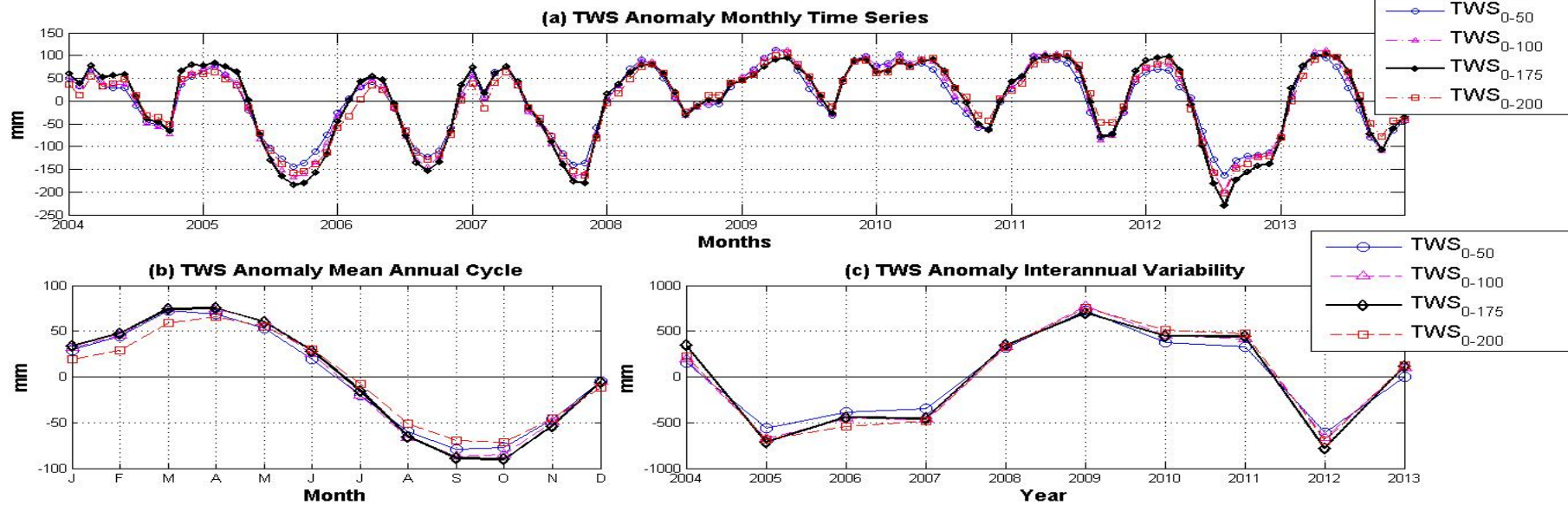


ID	Station Name	Latitude (°N)	Longitude (°W)	P (mm/yr)	SM (mm)	H (m)
BRW	Brownstown	38.95	88.96	1041.3	788.3	-0.86
DEK	De Kalb	41.84	88.85	927.4	649.9	-1.09
FAI	Fairfield	38.38	88.39	1110.1	737.8	-0.99
FRM	Belleville	38.52	89.84	1036.7	719.0	-1.35
LLC	Springfield	39.73	89.61	926.6	672.2	-1.74
MON	Monmouth	40.93	90.72	890.9	668.6	-3.10
OLN	Olney	38.74	88.10	1091.5	739.4	-0.88
ORR	Orr Center (Perry)	39.81	90.82	924.6	727.5	-2.47
RND	Rend Lake (Ina)	38.14	88.92	1130.2	741.4	-1.42
SIU	Carbondale	37.70	89.24	1192.2	709.7	-1.70
STC	St. Charles	41.90	88.36	830.0	641.1	-7.25
STE	Stelle	40.95	88.16	866.6	649.8	-1.01



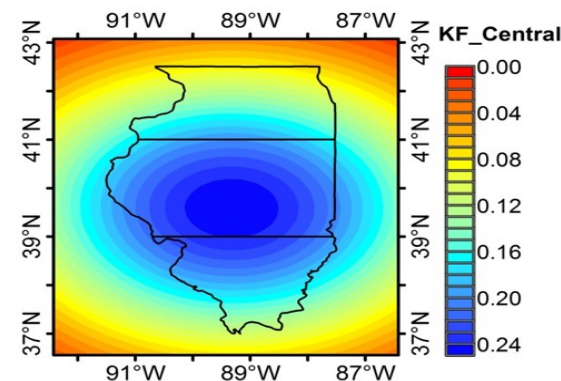
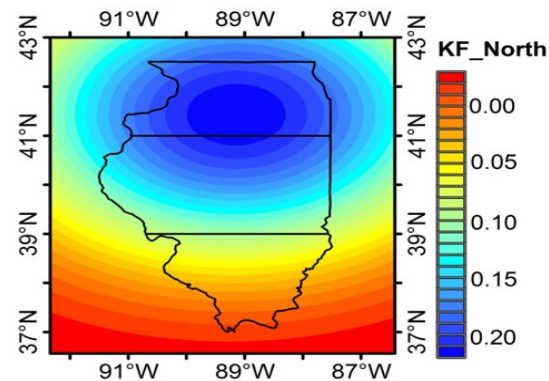
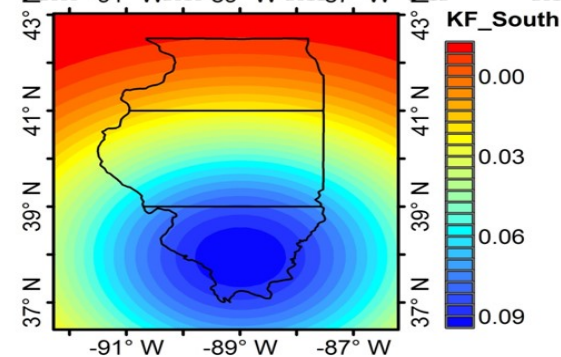
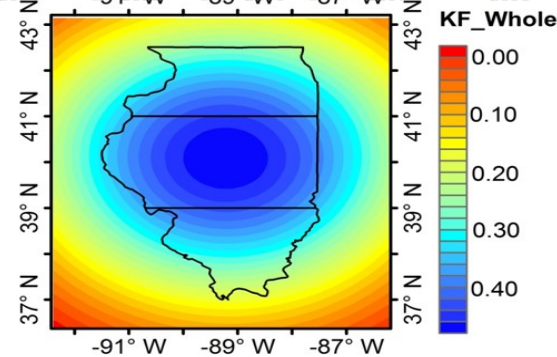
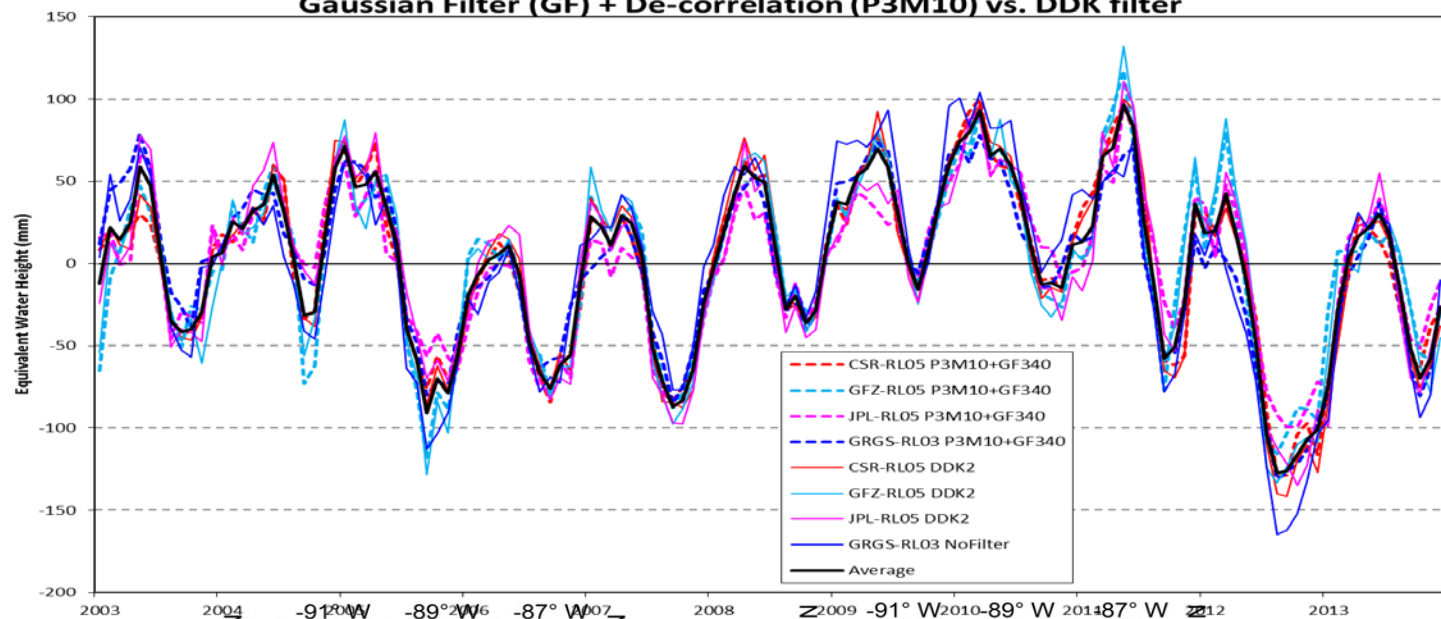
Illinois Soil Moisture and Groundwater Data

1. In this study a 30-year (1984-2013) monthly and daily hydrological dataset in Illinois is used to estimate regional GW recharge and baseflow by using soil water balance computation and multiple linear regressions.
2. Daily soil moisture data available only 2004-2013; linear interpolation based on bi-weekly SM measurements from 1984-2003 is used to obtain a 30-year daily SM series.
3. Daily GW (Water Table Table) data available only starts 1997; linear interpolation base on monthly GW data is used to obtain a 30-year daily GW data.
4. Precipitation data are averaged from 17 ISWS WARM stations; Streamflow averaged from 10 USGS station covering >80% of Illinois areas.

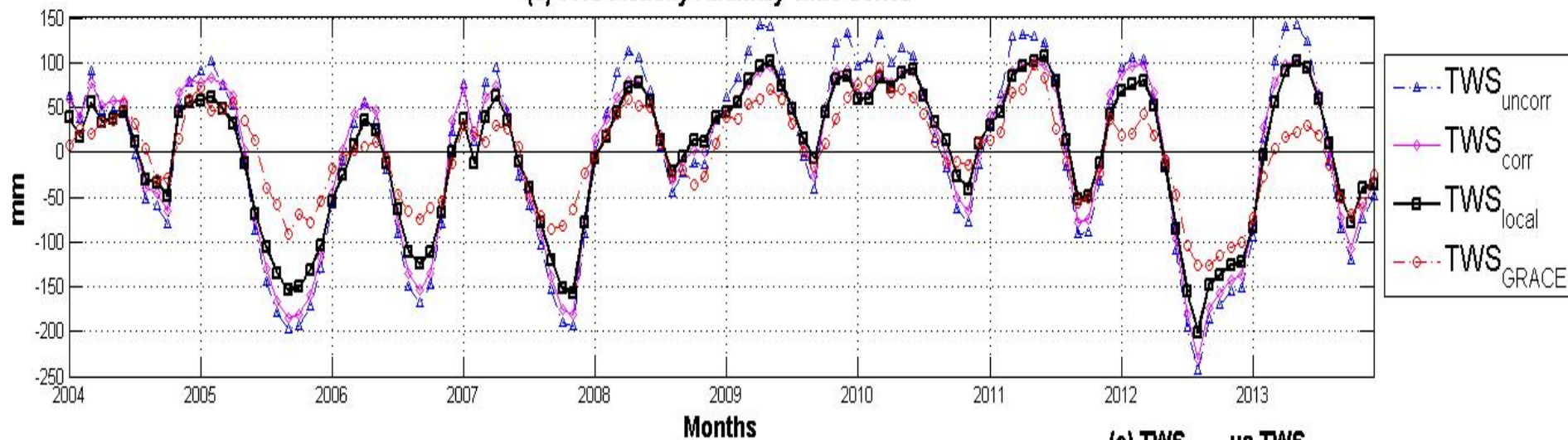


GRACE-derived terrestrial water storage variation in Illinois (2003-2013)

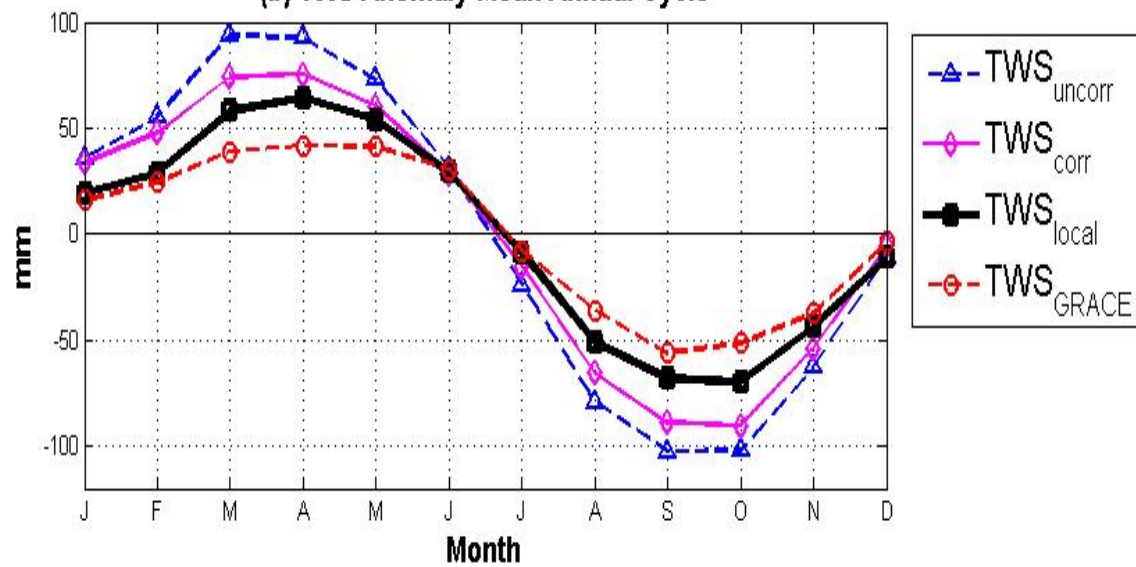
Gaussian Filter (GF) + De-correlation (P3M10) vs. DDK filter



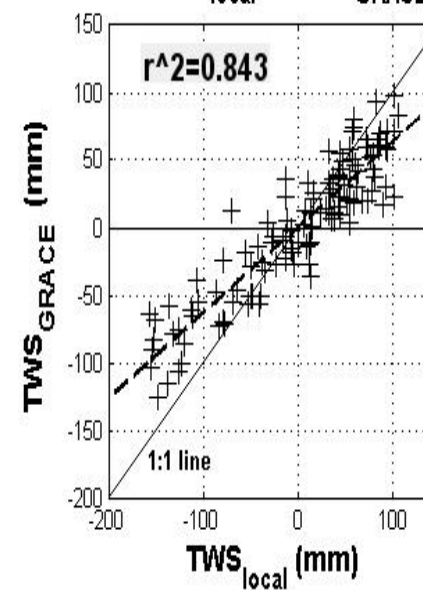
(a) TWS Monthly Anomaly Time Series



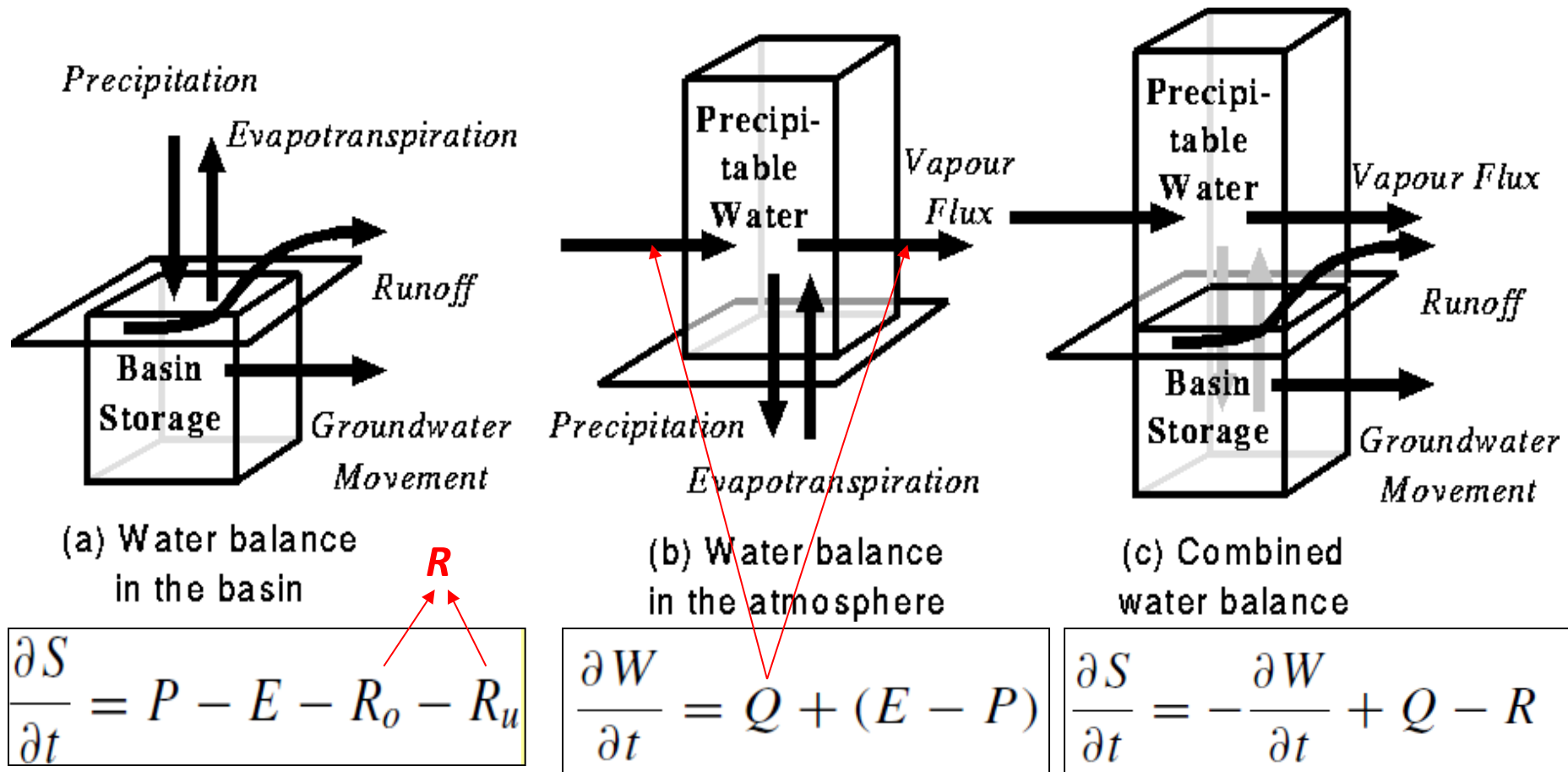
(b) TWS Anomaly Mean Annual Cycle



(c) TWS_{local} vs TWS_{GRACE}



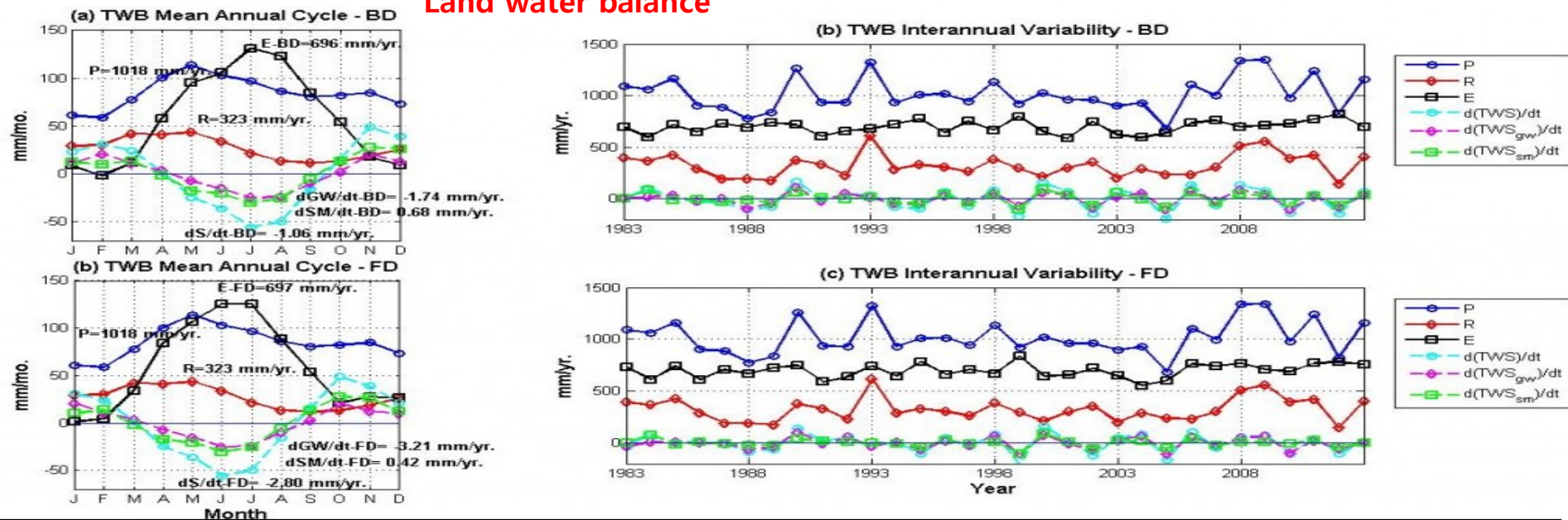
Land, Atmospheric, and Combined Water Balances



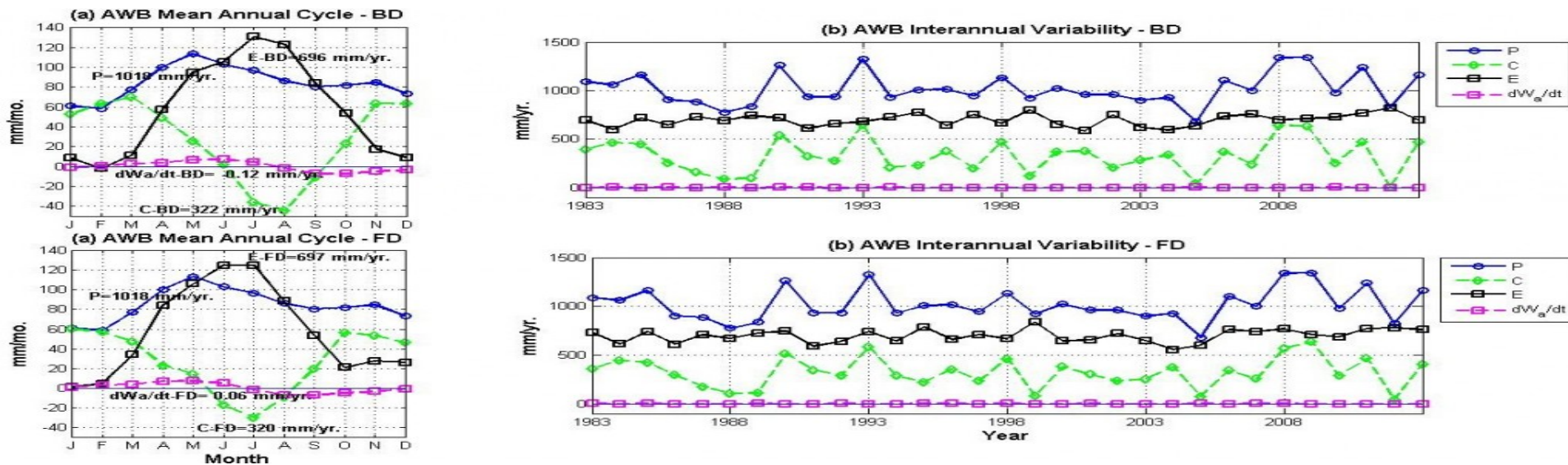
Relative change of total **terrestrial water storage (TWS)** can be estimated from water vapor convergence **Q** and river discharge **R** by using based on **water balance analysis** 8

1983-2013 Land and Atmospheric Water Balance

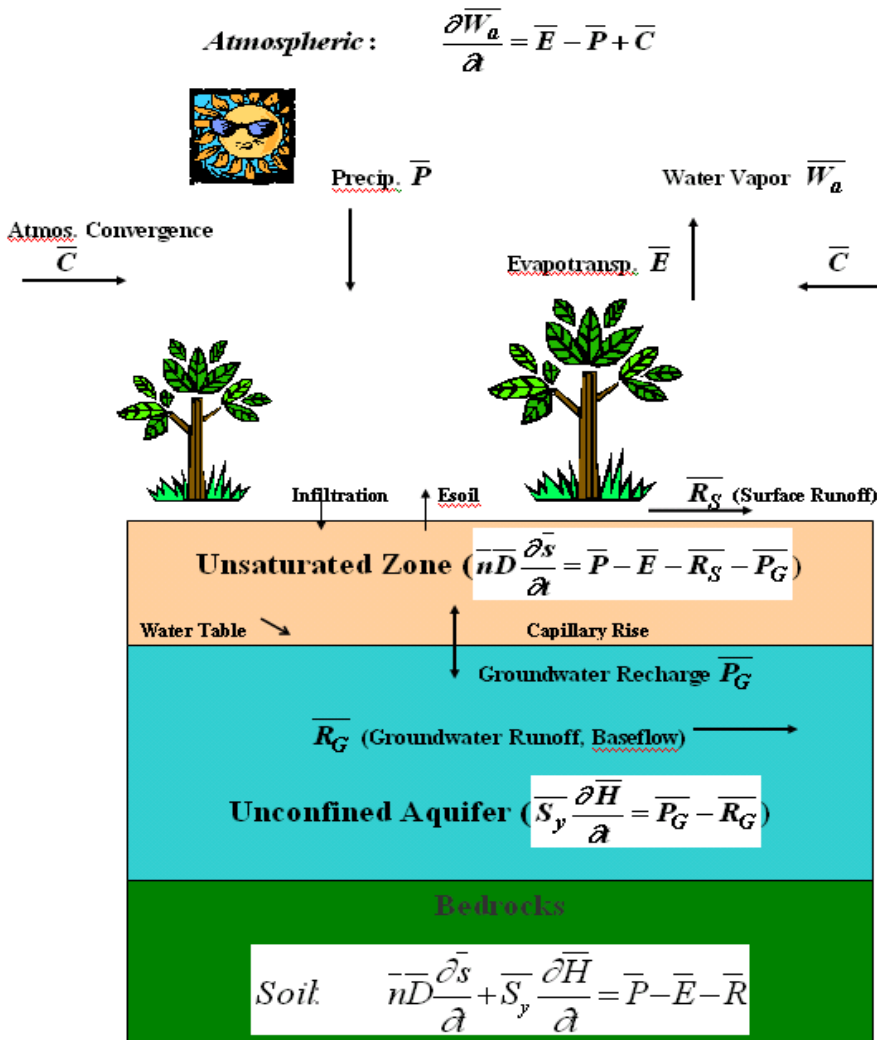
Land water balance



Atmospheric water balance



Terrestrial and Atmospheric Water Balance-



$$\bar{nD} \frac{d\bar{s}}{dt} + \bar{S}_y \frac{d\bar{H}}{dt} = \bar{P} - \bar{E} - \bar{R}$$

$$\bar{R} = \bar{R}_s + \bar{R}_g$$

$$\frac{d\bar{W}_a}{dt} = \bar{E} - \bar{P} + \bar{C}$$

Unsat. and Sat. Zone Water Balances

$$\bar{nD} \frac{d\bar{s}}{dt} = \bar{P} - \bar{E} - \bar{R}_s - \bar{P}_g$$

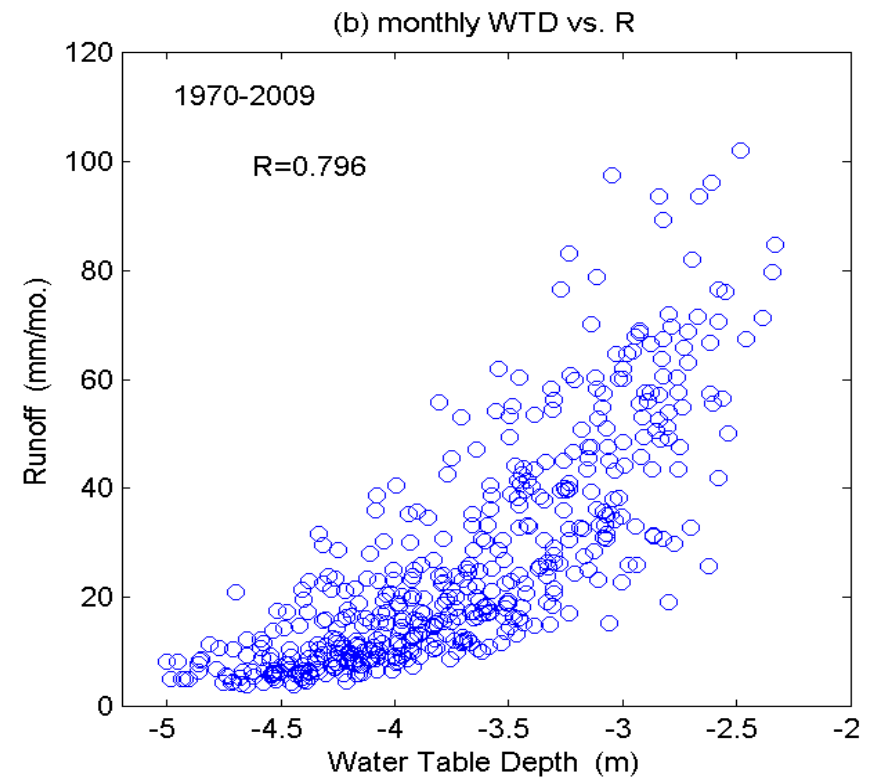
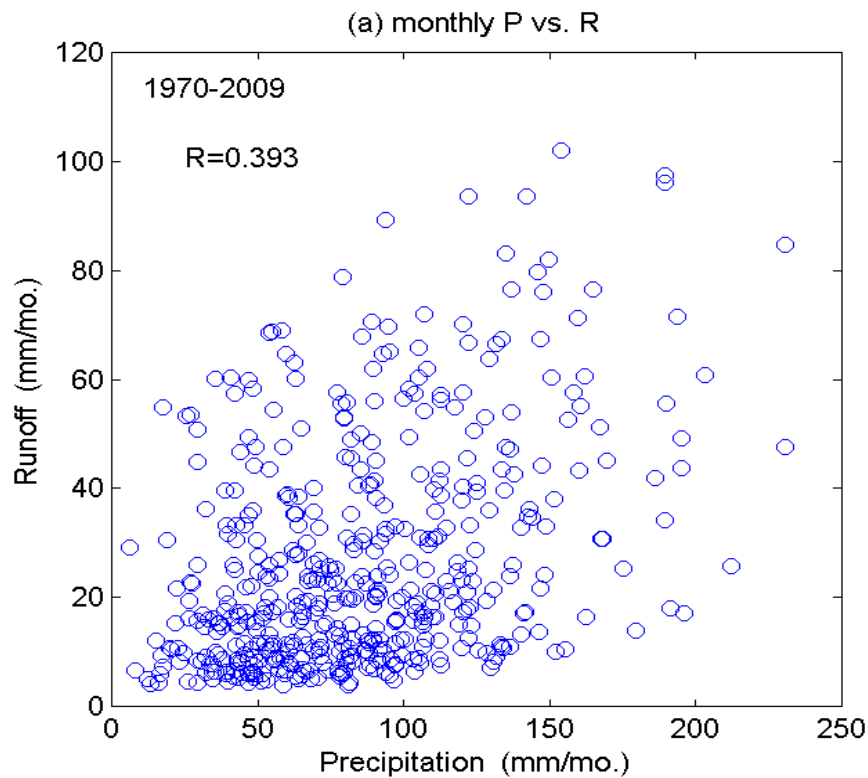
$$\bar{S}_y \frac{d\bar{H}}{dt} = \bar{P}_g - \bar{R}_g$$

Baseflow Separation based on:

1. multiple linear regression
2. digital recursive Filter

3 unknowns, but only 2 equations!!

Monthly Precip., Runoff, Water Table Depth in Illinois



Subsurface Water Storage Dominates Runoff Generation in Illinois

Regional Groundwater Evapotranspiration in Illinois

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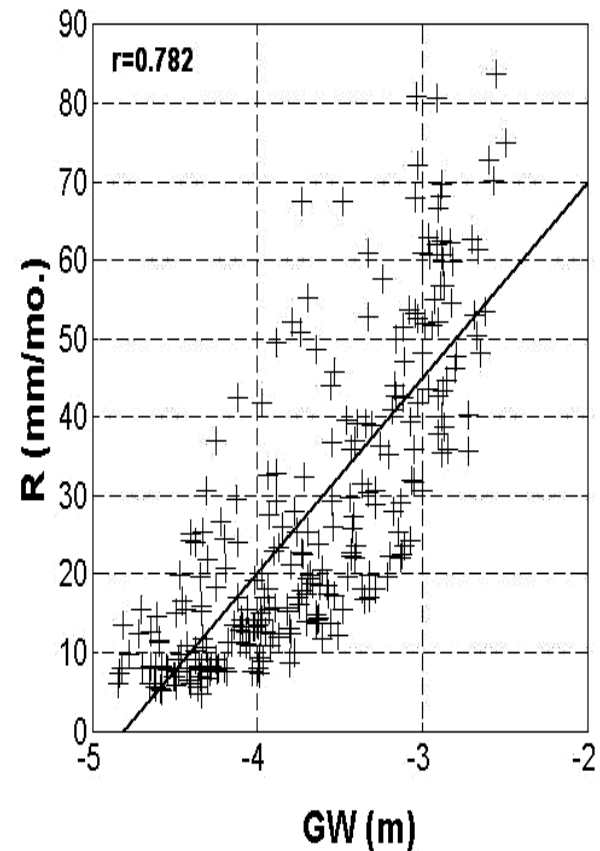
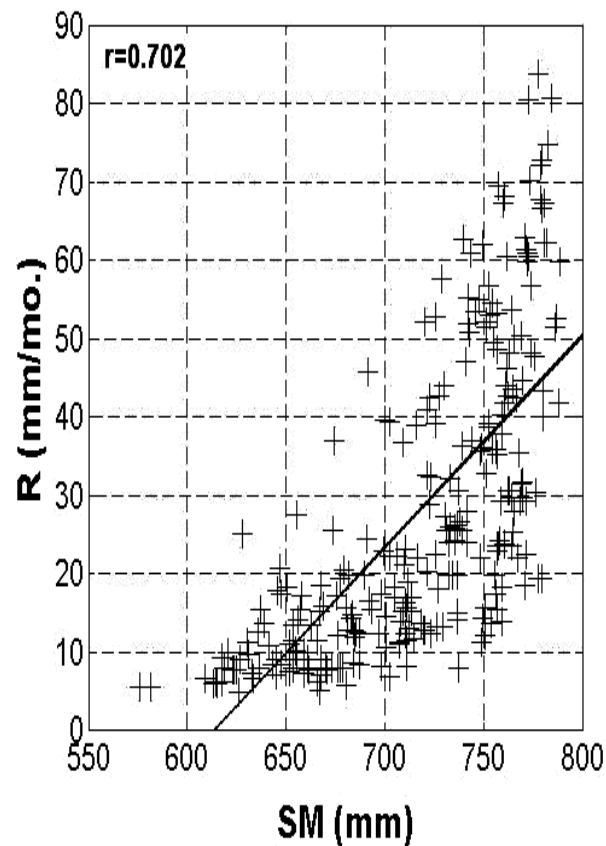
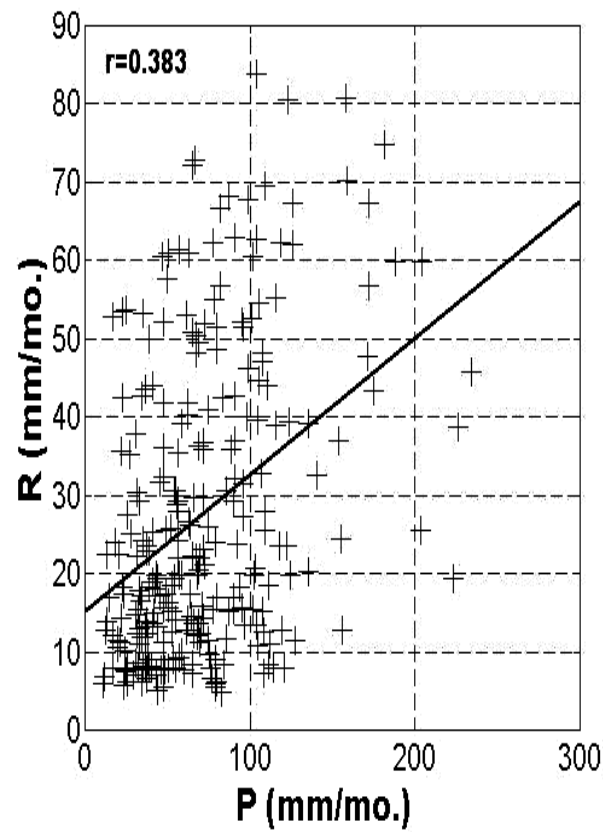
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(Manuscript received 13 February 2008, in final form 8 October 2008)

ABSTRACT

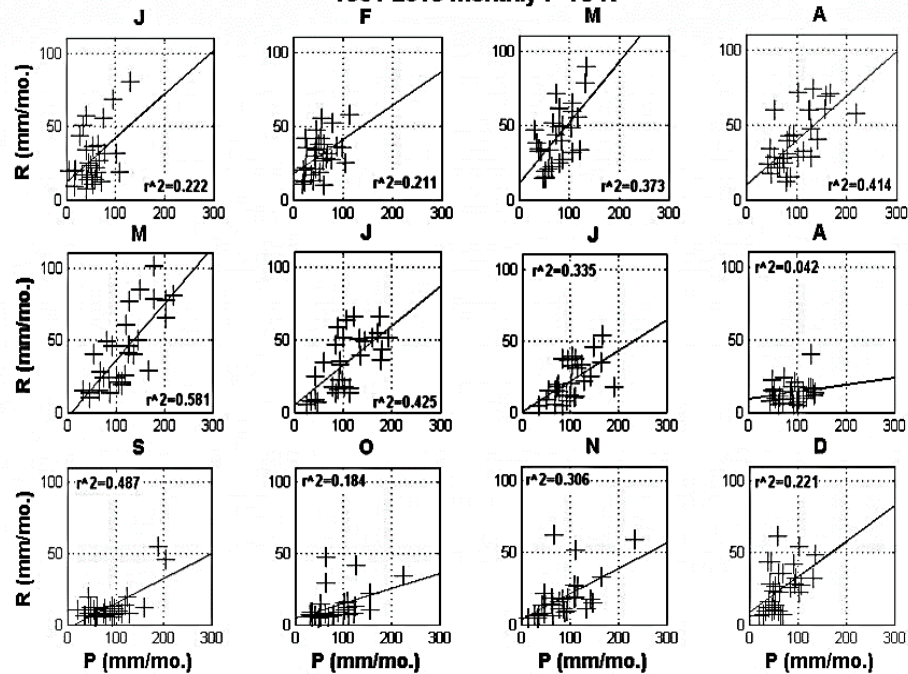
The role of shallow unconfined aquifers in supplying water for evapotranspiration (i.e., groundwater evaporation) is investigated in this paper. Recent results from regional land surface modeling have indicated that in shallow water table areas, a large portion of evapotranspiration comes directly from aquifers. However, little field evidence at the regional scale has been reported to support this finding. Using a comprehensive 19-yr (1984–2002) monthly hydrological dataset on soil moisture, water table depth, and streamflow in Illinois, regional recharge to and evaporation from groundwater are estimated by using soil water balance computation. The 19-yr mean groundwater recharge is estimated to be 244 mm yr^{-1} (25% of precipitation), with uncertainty ranging from 202 to 278 mm yr^{-1} . During the summer, the upward capillary flux from the shallow aquifer helps to maintain a high rate of evapotranspiration. Groundwater evaporation (negative groundwater recharge) occurs during the period of July–September, with a total of 31.4 mm (10% of evapotranspiration). Analysis of the relative soil saturation at 11 depths from 0 to 2 m deep supports the dominance of groundwater evaporation across the water table in dry periods. The zero-flux plane separating the recharge zone from the evapotranspiration zone propagates downward from about 70- to 110-cm depth during summer, reflecting the water supply from progressively lower layers for evapotranspiration. Despite its small magnitude, neglecting regional groundwater evaporation in shallow groundwater areas would result in underestimated root-zone soil moisture and hence evapotranspiration by as large as 20% in the dry summer seasons.



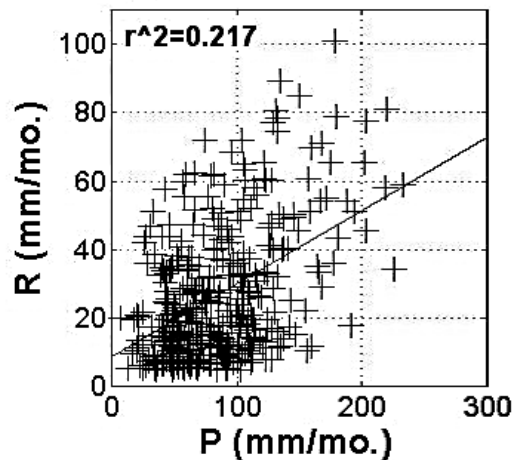
1. R can be decomposed to surface runoff, R_s (as a function of P), and baseflow, R_g (GW).
2. Only 15% of the variability in monthly discharge (R) can be explained by monthly precipitation (P). No clear P - R relationship is observed in P versus R scatter plot.
3. 49% and 61% of the variability of R can be explained by monthly soil moisture (SM) from 0-200cm soil depth and groundwater depth (GW) respectively. Thus, baseflow dominates R than surface runoff.
4. GW - R relationship is more non-linear. Regression of R will be performed using $\exp(GW)$.

Correlation among monthly R, P, SM

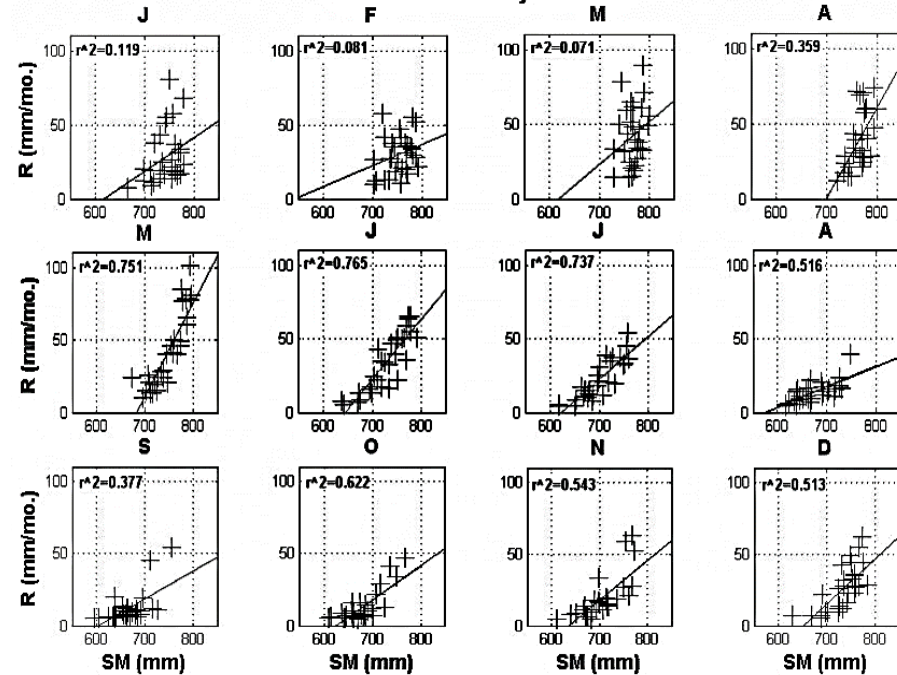
1984-2013 monthly P vs R



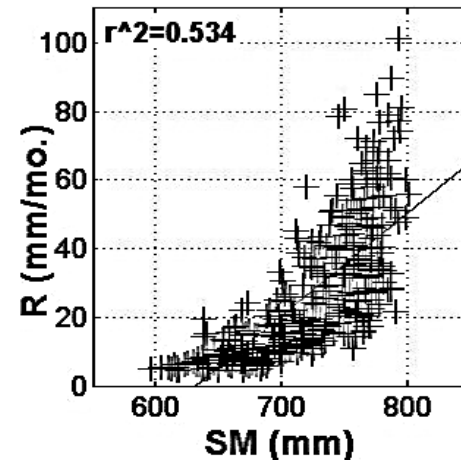
(a) 1984-13 monthly P vs R



1984-2013 monthly SM vs R

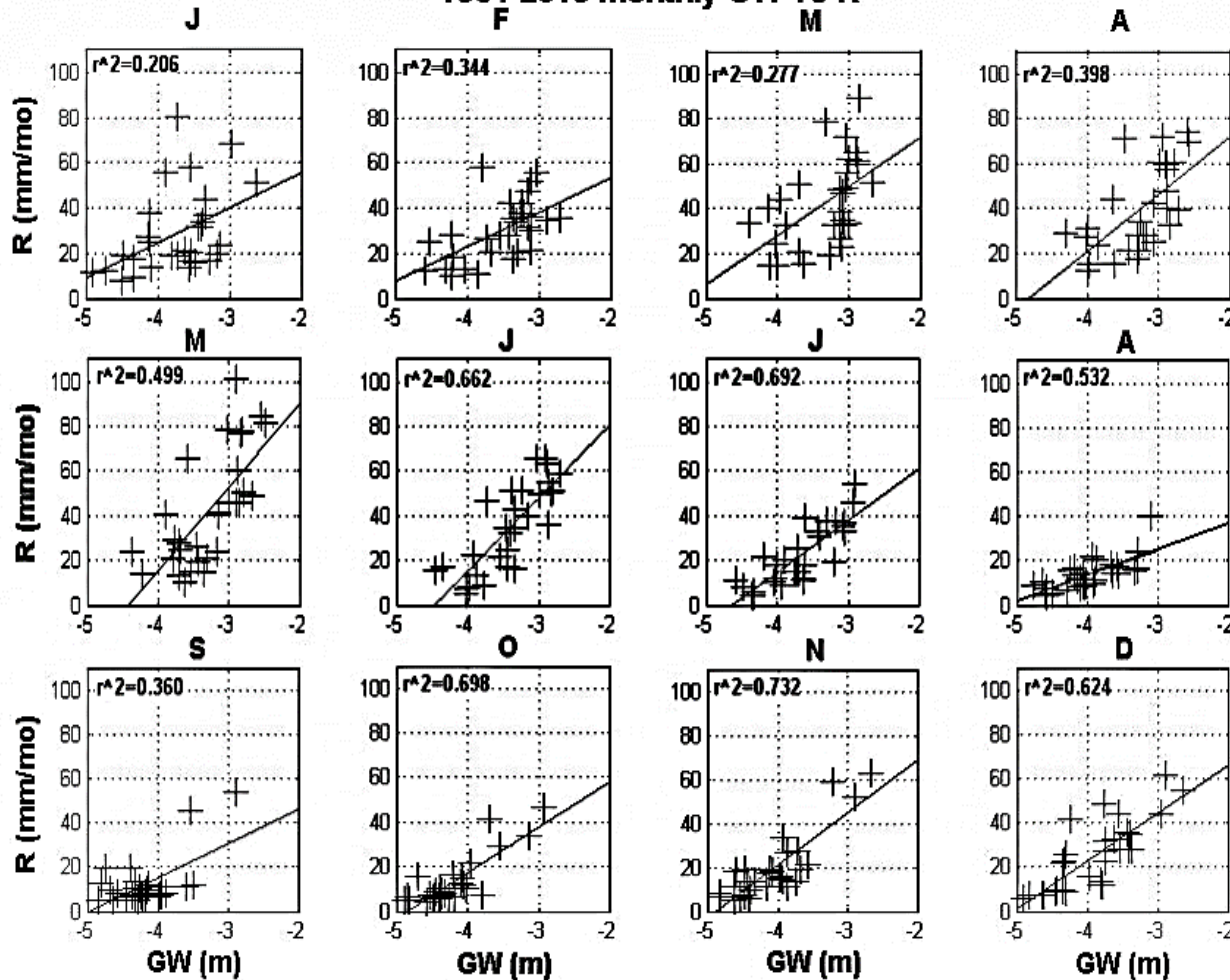


(b) 1984-13 monthly SM vs R

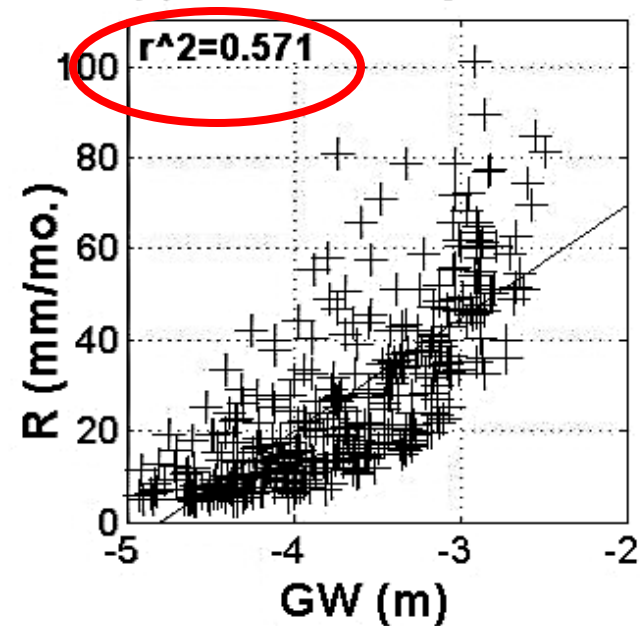


Correlation between monthly R and GW

1984-2013 monthly GW vs R



(c) 1984-13 monthly GW vs R



Monthly Multiple Linear Regression

Regression equation	R ² (average based on monthly regression)
$R = a * P + b * SM + c * \exp(GW) + d$	0.72
$R = a * P + b * \exp(GW) + c$	0.70
$R = a * P + b * SM + c * GW + d$	0.70
$R = a * P + b * GW + c$	0.67
$R = a * SM + b * \exp(GW) + c$	0.60
$R = a * P + b * SM + c$	0.58
$R = a * SM + b * GW + c$	0.56
$R = a * \exp(GW) + b$	0.55
$R = a * GW + b$	0.50
$R = a * SM + b$	0.45
$R = a * P + b$	0.32

Monthly Multiple Linear Regression

- $R = 0.12P + 0.14SM + 535.96 \cdot \exp(GW) - 99.46$, $R^2 = 0.72$
yields some negative monthly R during dry years 1988, 1999, 2005, 2012
- $R = 0.12P + 831.78 \cdot \exp(GW) - 6.84$, $R^2 = 0.70$, is chosen

$$R = a * P + b * \exp(GW) + c$$

Variable	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Constant
a	0.35	0.27	0.39	0.19	0.28	0.08	0.10	0.03	0.11	0.04	0.11	0.17	0.12
b	675.08	588.57	684.71	529.10	701.69	835.24	770.25	581.96	647.83	907.69	884.61	761.83	831.78
c	-8.42	-3.67	-14.25	-2.08	-19.03	-4.89	-10.15	-1.32	-8.38	-5.00	-8.36	-4.73	-6.84
R²	0.49	0.59	0.66	0.60	0.79	0.70	0.81	0.62	0.73	0.78	0.92	0.72	0.70
F-stat	12.93	19.64	26.29	20.27	51.43	31.10	58.05	21.72	37.38	47.30	163.21	35.31	376.61
p	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
error var.	182.33	75.84	140.57	157.41	145.79	122.32	35.75	20.77	34.15	28.91	18.31	72.04	122.05

Detailed Procedures of Regress Analyses

1. The form of Regression equation of total runoff is determined as:

$$R = a * P + b * \exp(GW) + c \quad (\text{GW defined as negative – the depth of water table})$$

2. 4 cases are considered: Regression with respect to (A) the entire 360 monthly data together, (B) 30 monthly data for each of the twelve months of a year; (C) the entire 10958 daily data all together; (D) the daily data for each of the twelve months of a year. Analysis (below) indicates (D) is the best to be used. (Note: for (c) and (D) daily analysis, P is the 21-day antecedent accumulated precipitation.)
3. In order to yield fully partitioning between surface runoff and baseflow, the regression constant c is re-written as

$$R = a * (P - P_0) + b * \exp(GW) = R_s + R_g$$

4. To avoid the occurrence of negative surface runoff R_s , the above regression equation (D) is further revised as

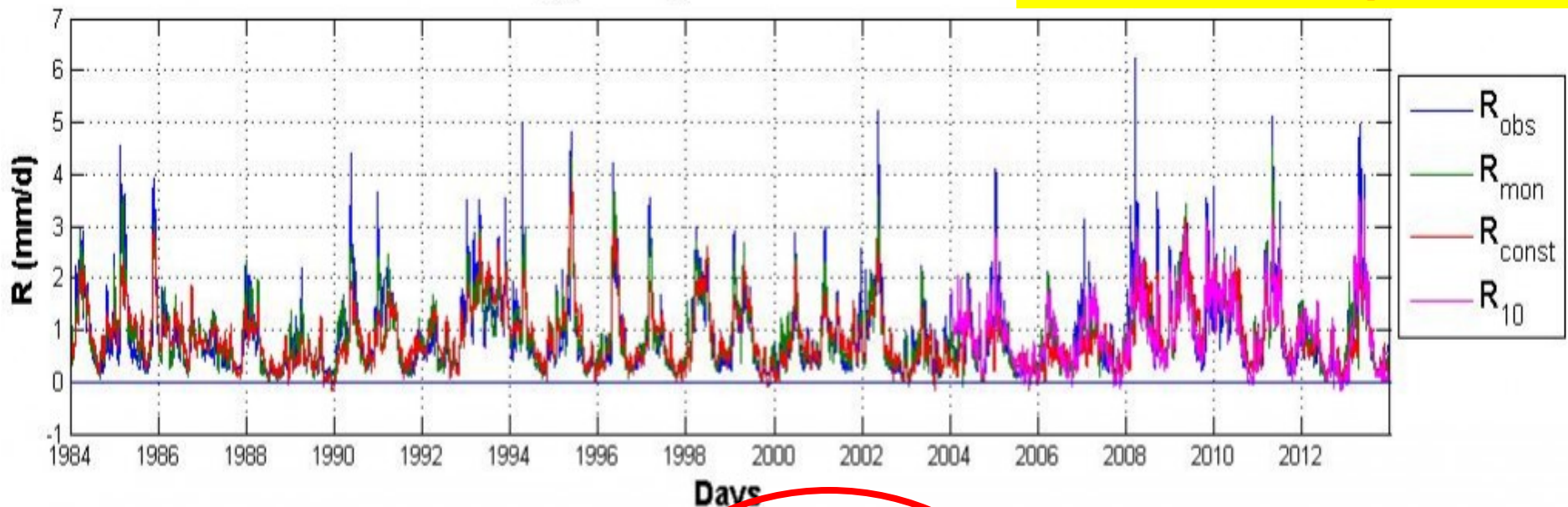
$$R = a * (P - P_0) + b * \exp(GW) = R_s + R_g \quad \text{if } P > P_0$$

$$R = b * \exp(GW) = R_g \quad \text{if } P \leq P_0$$

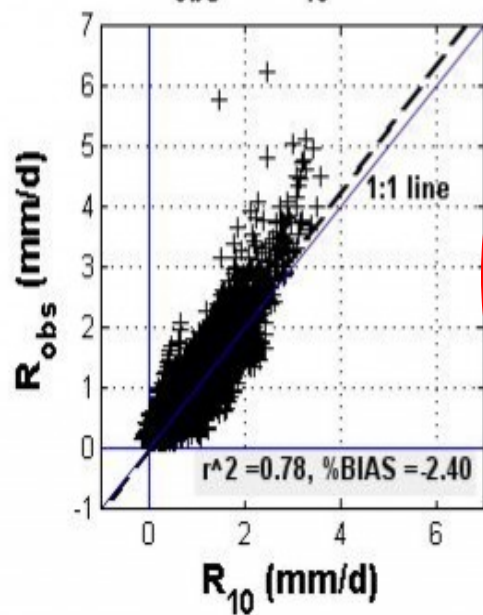
this would make more physical sense but potentially cause biased mean total runoff estimate; however, the occurrence of $P \leq P_0$ is rare, therefore the mean of fitted total runoff is very close to mean observed runoff.

(a) R_{obs} and $R_{fit}(P, \exp(GW))$ (1984-13)

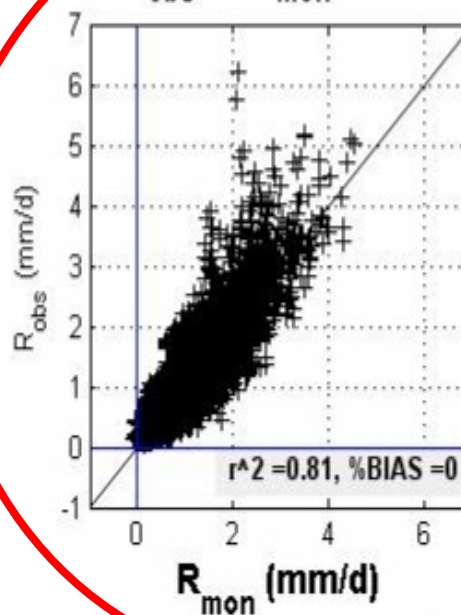
$$R = a \cdot P + b \cdot \exp(GW) + c$$



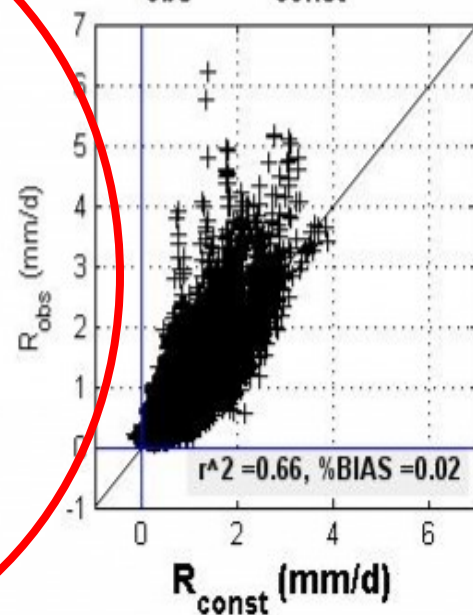
(b) R_{obs} vs R_{10} (2004-13)



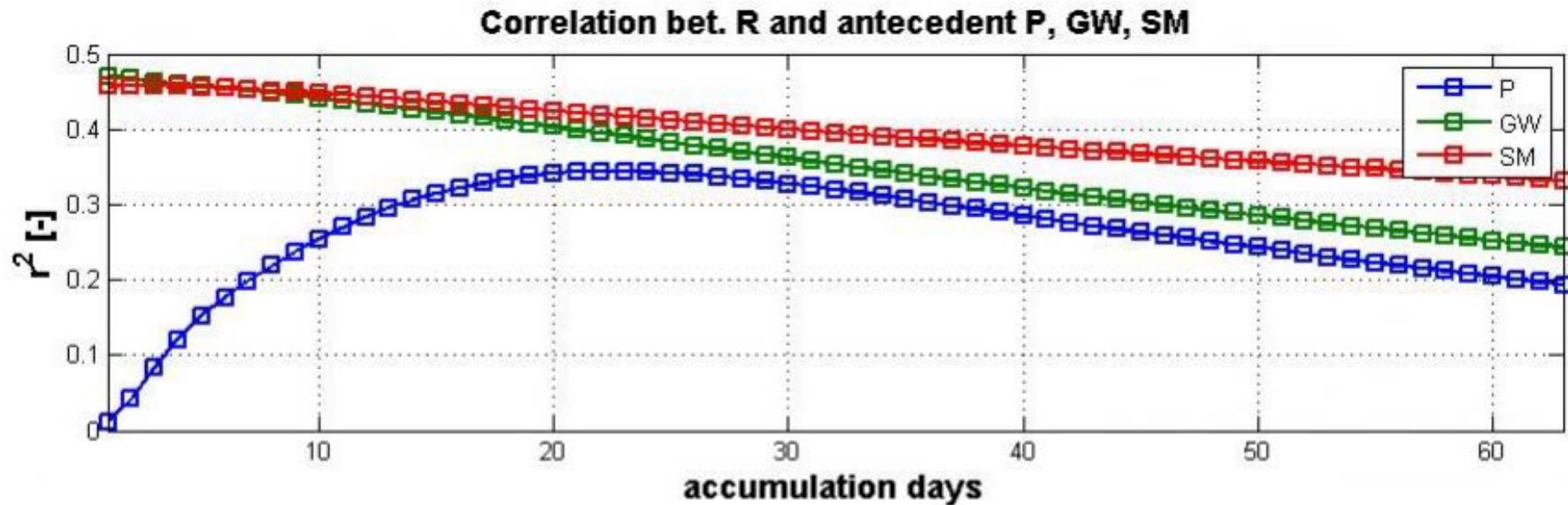
(c) R_{obs} vs R_{mon} (1984-13)



(d) R_{obs} vs R_{const} (1984-13)

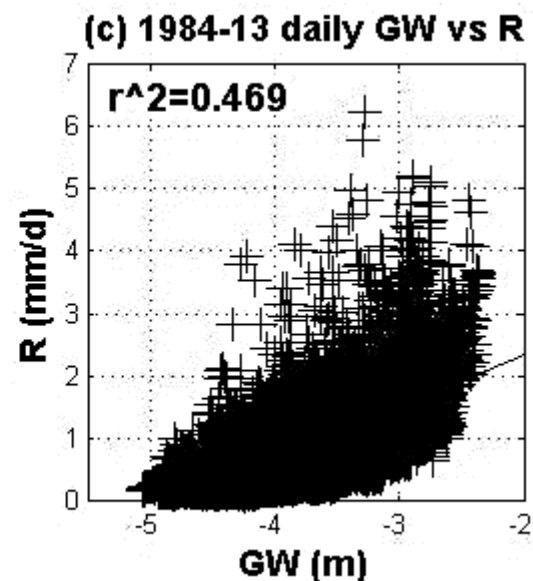
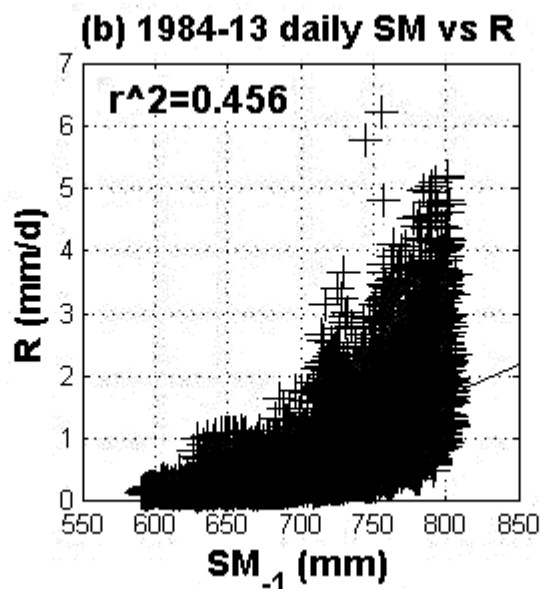
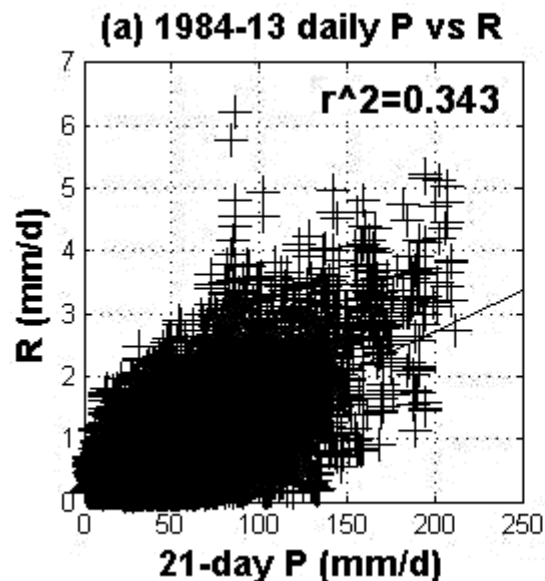


Daily Correlation between R and P_{cum} , SM and GW



Lag of R behind P, GW and SM is evaluated by finding the maximum correlation between present-day R with cumulative antecedent P (sum), SM and GW the number of days

Correlation between Daily R vs P, SM and GW



Highest $r^2=0.34$ was obtained from using 22-day accumulated P. So, **21-day P ($r^2=0.343$)** was used

1-day prior SM was used for the present-day R regression.

0-day lag between R and GW for maximum correlation

Daily Regression

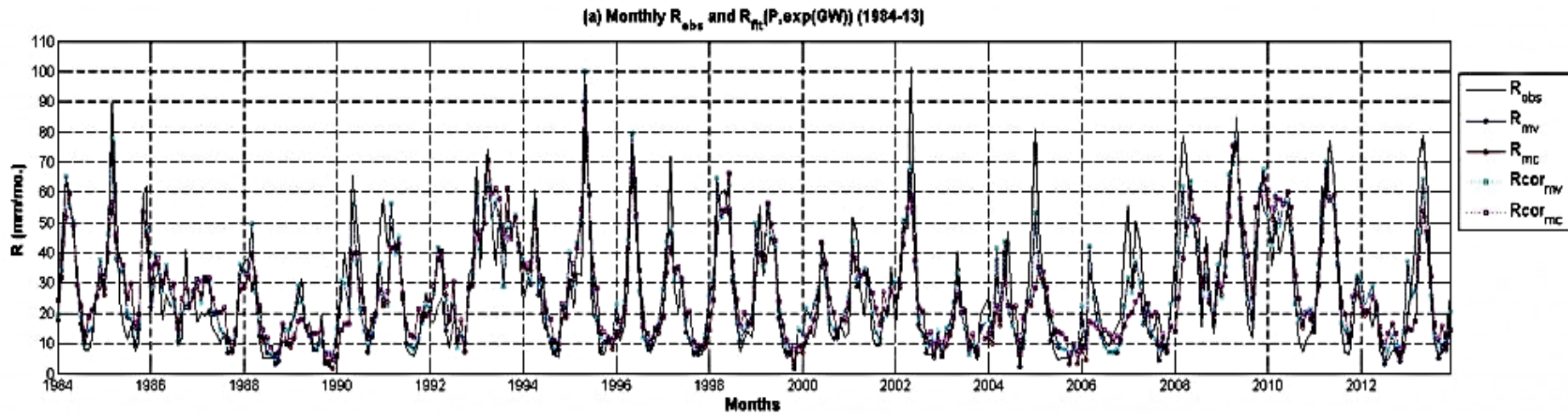
Equation	R ² (average based on daily regression)
$R = a * P + b * SM + c * \exp(GW) + d$	0.74
$R = a * P + b * \exp(GW) + c$	0.73
$R = a * P + b * SM + c * GW + d$	0.72
$R = a * P + b * GW + c$	0.71
$R = a * P + b * SM + c$	0.64
$R = a * P + b$	0.51
$R = a * SM + b * \exp(GW) + c$	0.50
$R = a * SM + b * GW + c$	0.46
$R = a * \exp(GW) + b$	0.45
$R = a * GW + b$	0.40
$R = a * SM + b$	0.37

Fitted R sometimes have negative values. Correction is madd by -

$$R = a * (P - P_0) + b * \exp(GW) = R_s + R_g \quad \text{if } P > P_0$$

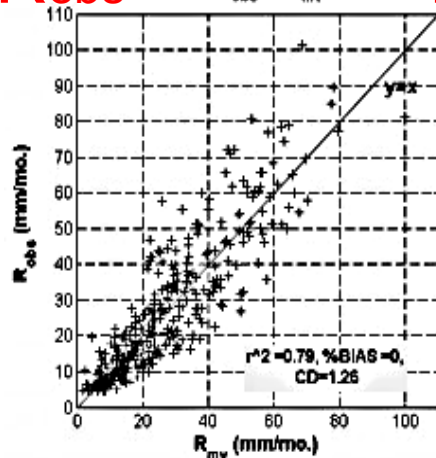
$$R = b * \exp(GW) = R_g \quad \text{if } P \leq P_0$$

Monthly R Estimation from Regression



R_{obs}

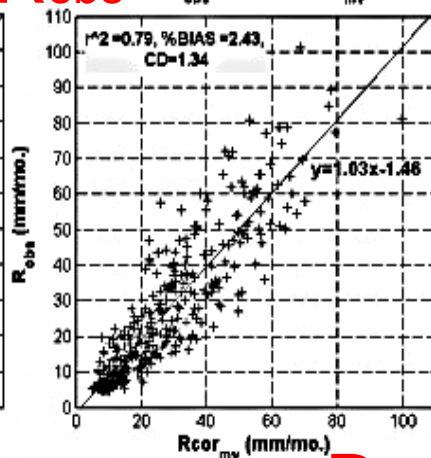
(b) R_{obs} vs R_{mv}



R_{mv}

R_{obs}

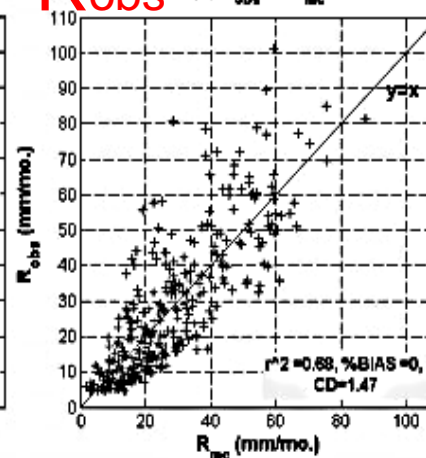
(c) R_{obs} vs corrected R_{mv}



Corrected R_{mv}

R_{obs}

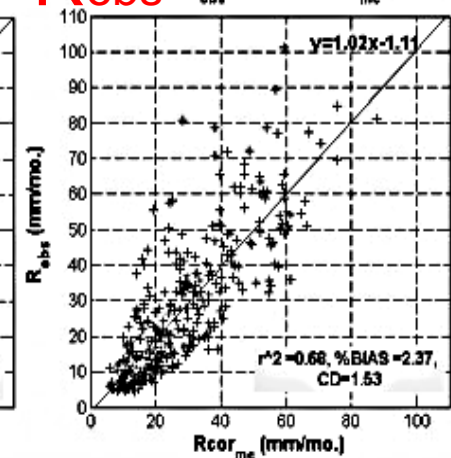
(d) R_{obs} vs R_{mc}



R_{mc}

R_{obs}

(e) R_{obs} vs corrected R_{mc}

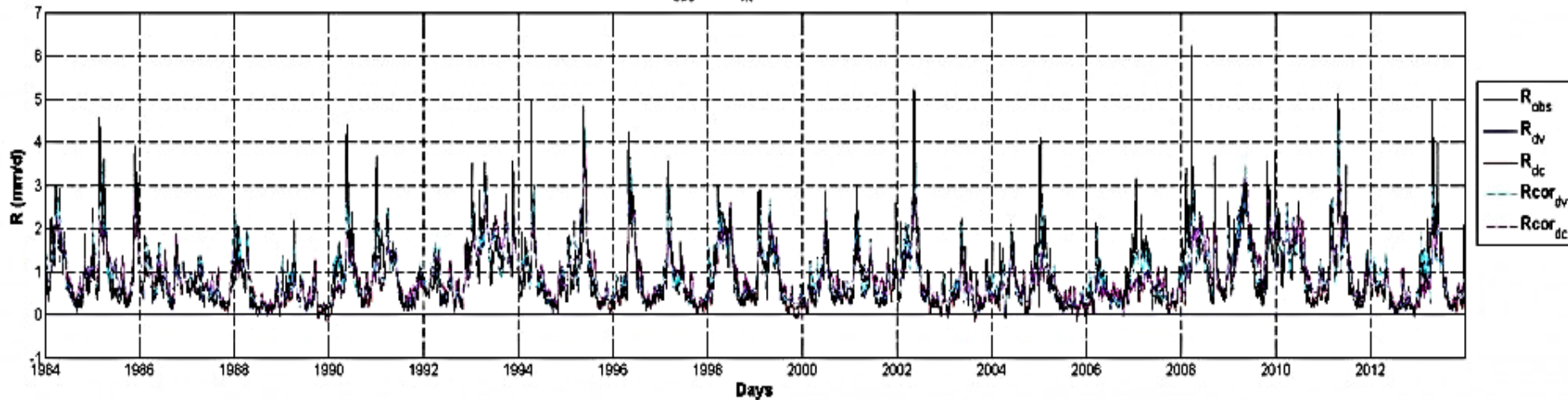


Corrected R_{mc}

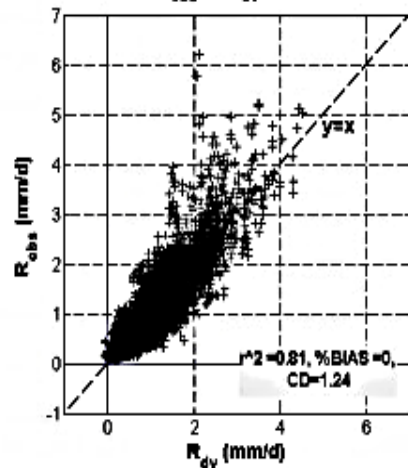
R_{mv} = Monthly regression with variable coefficients for each month
 R_{mc} = Monthly regression with constant coefficients for each month
 Corrected $R = a (P - P_0) + b * \exp(GW)$

Daily R Estimation from Regression

(a) Daily R_{obs} and $R_{fit}(P, \exp(GW))$ (1984-13)

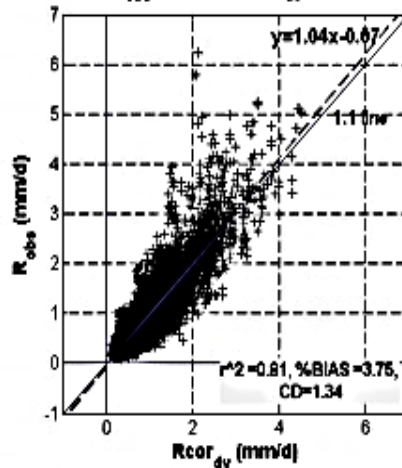


R_{obs} (b) R_{obs} vs R_{dv} (1984-13)



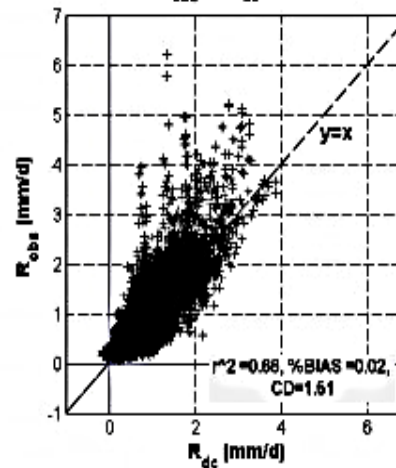
R_{dv}

R_{obs} (c) R_{obs} vs corrected R_{dv} (1984-13)



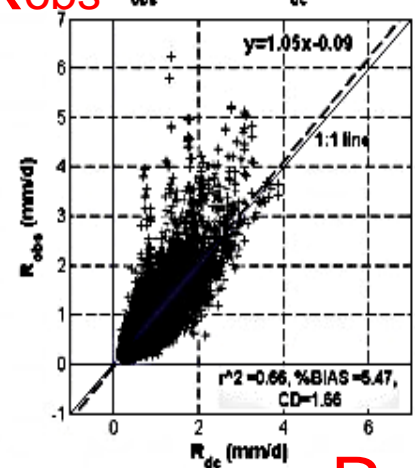
Corrected R_{dv}

R_{obs} (d) R_{obs} vs R_{dc} (1984-13)



R_{dc}

R_{obs} (e) R_{obs} vs corrected R_{dc} (1984-13)



Corrected R_{dc}

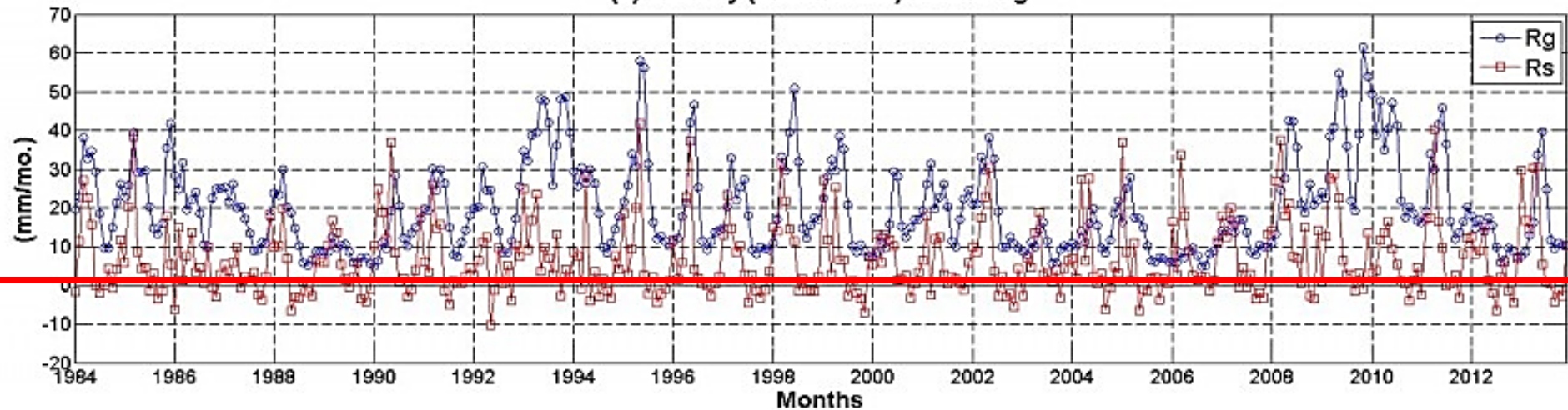
R_{dv} = Daily regression with variable coefficients for each month

R_{dc} = Daily regression with constant coefficients for each month

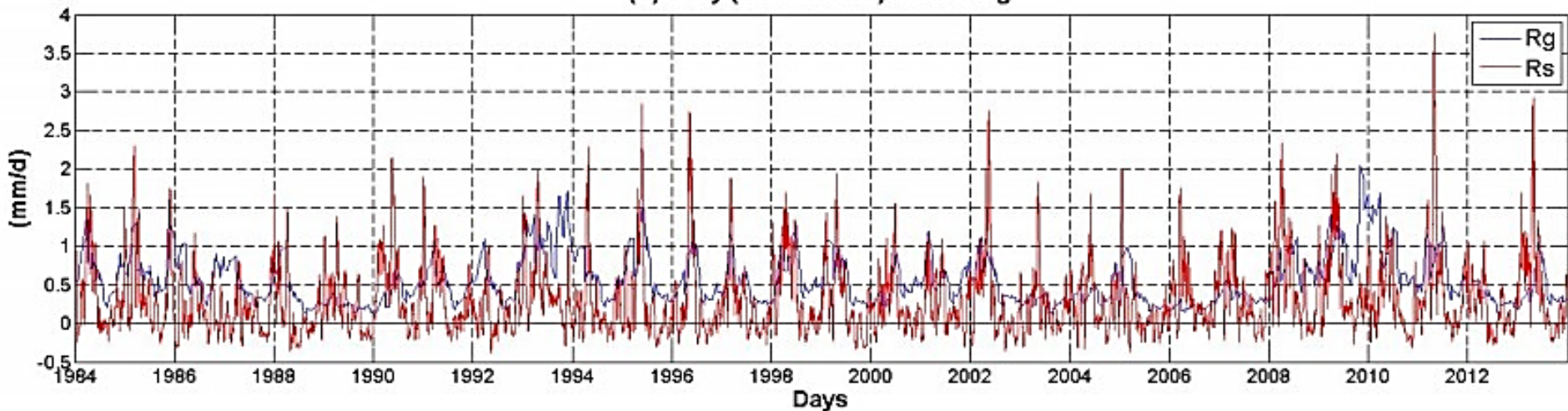
$$\text{Corrected R} = a (P - P_0) + b * \exp(GW)$$

Separation Between Baseflow and Surface Runoff

(a) Monthly (uncorrected) Rs and Rg



(b) Daily (uncorrected) Rs and Rg



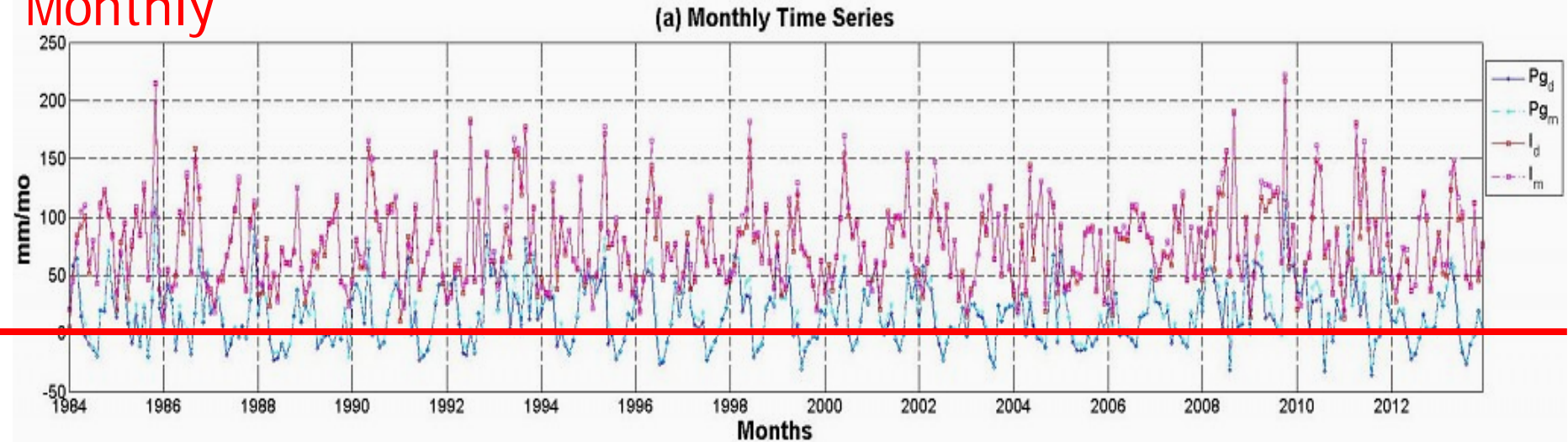
Some unrealistic negative surface runoffs remain for both monthly and daily regressions...

Statistics for Correction by using P0

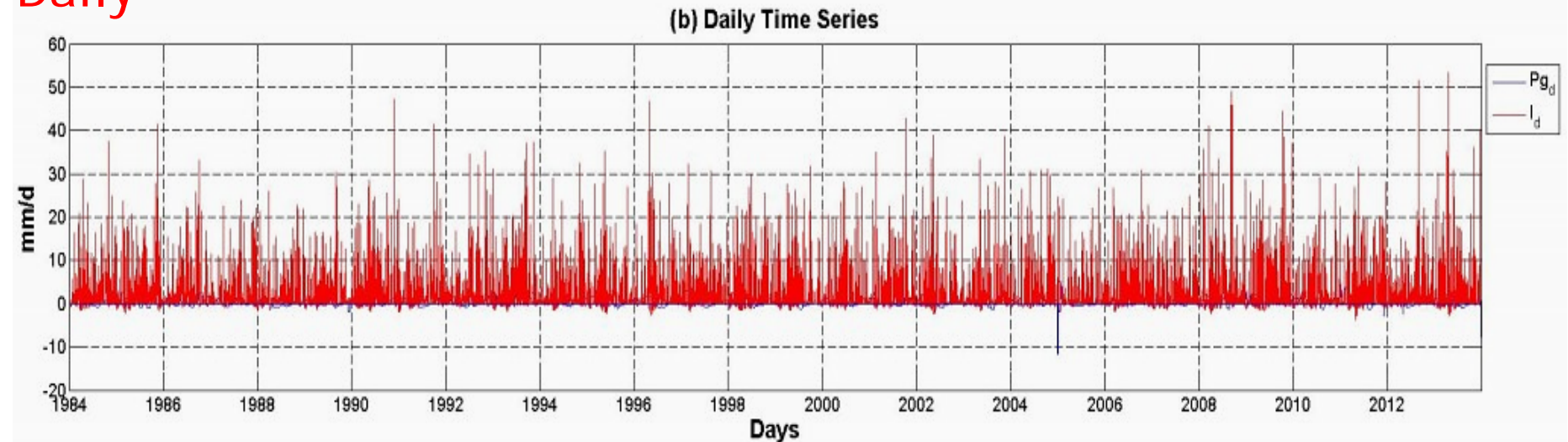
Rcorr=a(P-P0)+b*exp(GW)	daily (10958 days)				monthly (360 months)			
	alpha	P0	beta	no. of P < P0	alpha	P0	beta	no. of P < P0
JAN	0.019	18.79	18.713	215	0.348	24.17	675.079	4
FEB	0.019	18.77	22.190	197	0.270	13.60	588.569	0
MAR	0.021	23.48	22.159	130	0.394	36.20	684.709	2
APR	0.020	22.78	12.183	50	0.194	10.71	529.098	0
MAY	0.017	35.82	15.697	102	0.277	68.65	701.692	6
JUN	0.012	38.72	18.984	143	0.084	58.09	835.241	5
JUL	0.007	58.95	22.530	386	0.104	97.38	770.249	15
AUG	0.005	34.63	14.799	92	0.034	39.27	581.962	0
SEP	0.006	58.92	23.672	547	0.115	73.11	647.825	15
OCT	0.005	55.29	24.920	531	0.036	139.92	907.694	27
NOV	0.007	49.60	28.059	412	0.113	73.86	884.605	13
DEC	0.011	23.85	22.002	156	0.175	27.09	761.834	1
TOTAL/AVE.	0.01	36.63	20.49	2961	0.18	55.17	714.05	88
CONSTANT	0.009	42.78	25.761	3946	0.116	58.82	831.778	114

Infiltration and GW Recharge Comparison

Monthly



Daily

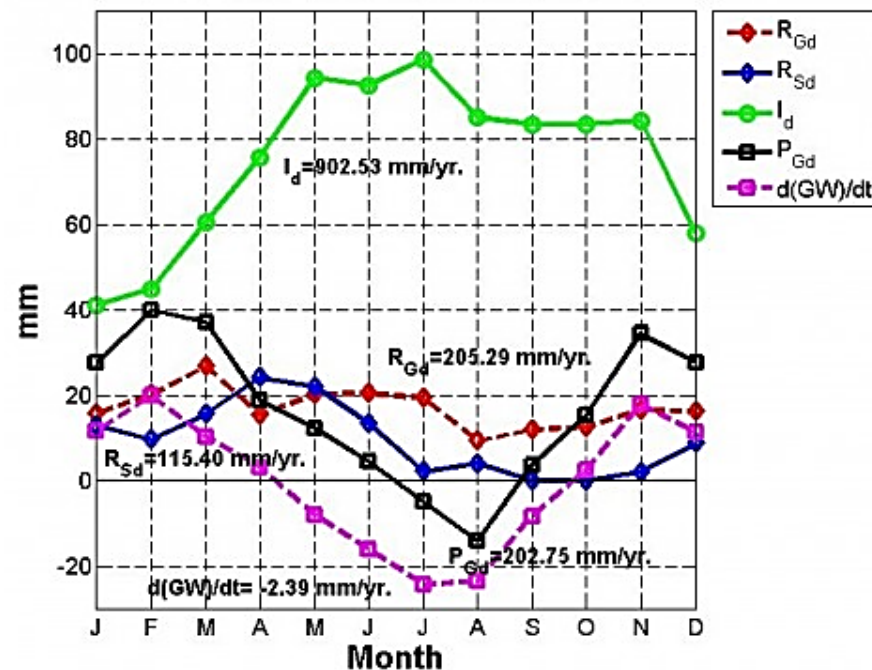


Some Negative GW Recharge in Summer Represents GW Evaporation

Mean seasonal cycles of soil water balance components

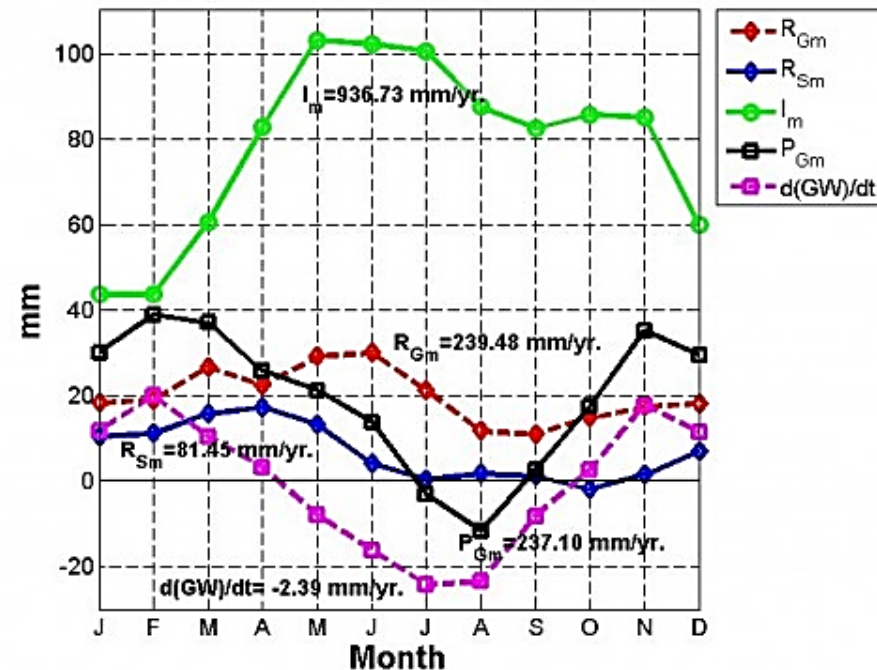
From Daily Regression

(a) Daily Soil Water Balance



From Monthly Regression

(b) Monthly Soil Water Balance



Summary for Part I Study

- 1984-2013 mean observed $R = 320.93$ mm/yr. in Illinois. Surface runoff R_s is estimated between 36-81 mm/yr. based on monthly analysis; 51-115 mm/yr. based on daily analysis.
- The partitioning between R_s and baseflow R_g is sensitive to analysis time scale - Daily analysis is necessary to resolve important Precip- R_s interactions even GW is interpolated.
- R_s follows seasonal SM and GW pattern. R_g is more constant over the year
- R_g is 239 to 284 mm/yr (monthly); 205 to 269 mm/yr (daily)
- GW recharge was 237 to 282 mm/yr (monthly dataset); 202 to 266 mm/yr (daily dataset). Long-term mean R_g balances GW recharge.
- GW recharge is high from Nov through March, turns slightly negative in July and August due to GW Evaporation.

Regionalized Drought Flow Hydrographs From a Mature Glaciated Plateau

WILFRIED BRUTSAERT AND JOHN L. NIEBER

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The drought or base flow characteristics of six basins in the Finger Lakes region are obtained by considering for each available record the lower envelope of $|dQ/dt|$ as a function of Q , where Q is the flow rate. This procedure avoids the uncertainty regarding a proper time reference after each rainfall event, and it eliminates the effects of evapotranspiration. The results suggest that among several expressions, Boussinesq's nonlinear solution of free surface groundwater flow is best suited to parameterize the observed hydrographs. The obtained parameters can be related to the basin characteristics, viz., drainage area and the total stream length, in accordance with relationships derived on the basis of the Dupuit-Boussinesq aquifer model. This result allows the determination of drought flow parameters for ungaged sites within the region.

WATER RESOURCES RESEARCH, VOL. 44, W02409, doi:10.1029/2007WR006518, 2008



Long-term groundwater storage trends estimated from streamflow records: Climatic perspective

Wilfried Brutsaert¹

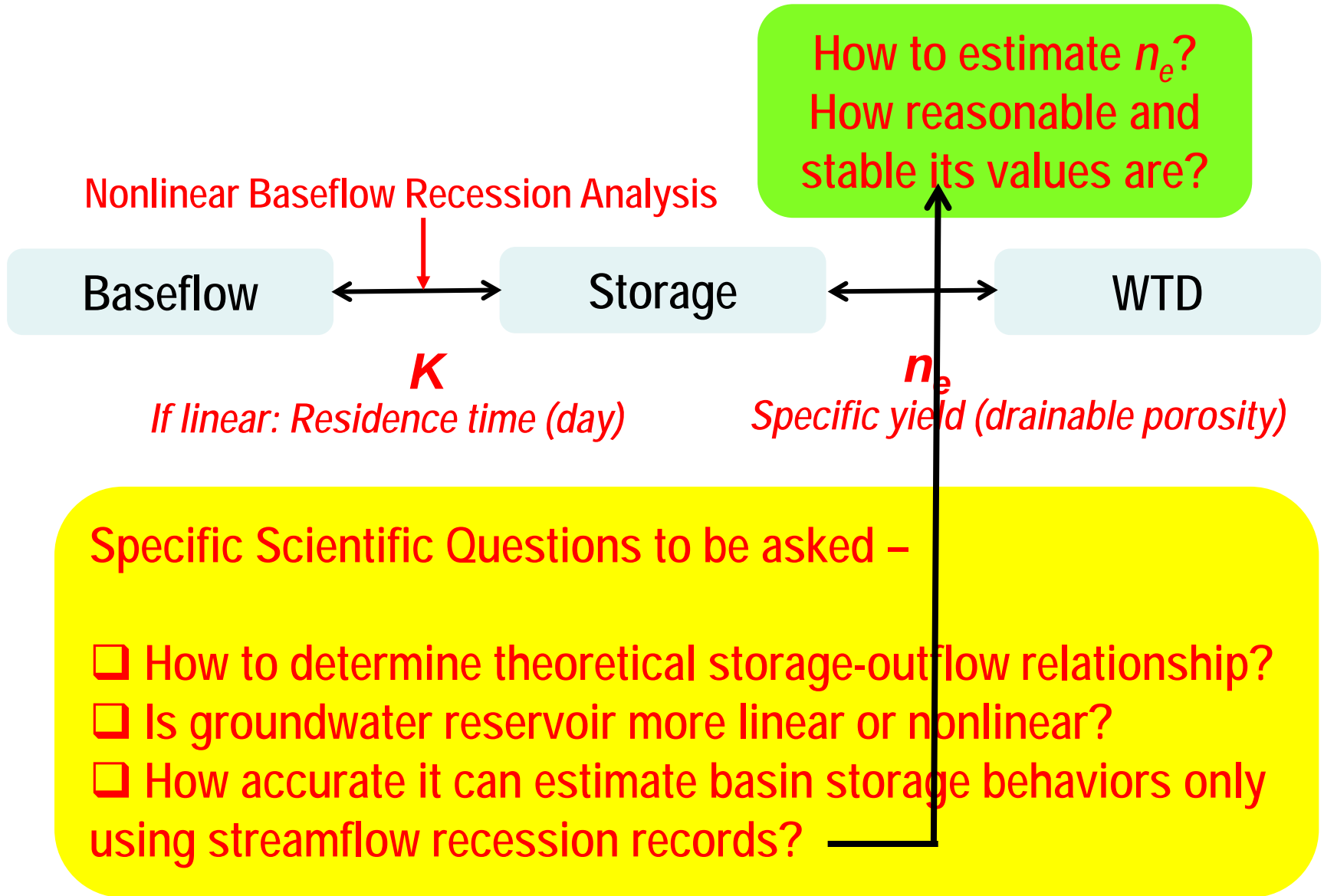
Received 12 September 2007; revised 28 October 2007; accepted 5 November 2007; published 5 February 2008.

[1] Base flow, also known as low flow or drought flow, in a natural river system originates mainly from drainage of the riparian aquifers in the upstream basin. Base flow data can therefore provide a quantitative estimate of the basin-scale groundwater storage changes that have taken place over the period of record. The concept was implemented and validated with data from two large river basins in Illinois. On average over the past 2/3 century, shallow groundwater storage in Illinois, derived from the drought flows, has been increasing at an average rate of around 0.05 to 0.10 mm a^{-1} .
This study is part of a special issue entitled "Groundwater and the Environment," which is part of the "Special Issue on Groundwater and the Environment," published in the June 2008 issue of the *Journal of Hydrology*.

Motivation: Brutsaert (2008, WRR)

- *Brutsaert* (2008, WRR, Table 2) conducted a linear analysis, compared the estimated 1965-2000 trend of groundwater storage (from baseflow recession) with the trend of annual lowest water table depth (WTD), and found good correlation of 0.66 between them. Aquifer residence time $\sim 45 \pm 15$ days.
- He further validated the estimates by calculating the drainable porosity (specific yield) n_e , and found, in average, 0.05835 for Illinois River Basin, and 0.01434 for Rock River Basin (since the WTD trend in Illinois basins is much smaller than Rock basin). For the Illinois average, $n_e = 0.01318$, and $= 0.00897$ if annual averaged WTD is used instead of the lowest WTD.
- He compared with the slope of Baseflow-WTD relation by *Eltahir and Yeh* (1999) and found consistency with the smaller n_e as another support to his estimate.
- He commented these n_e estimates much smaller than the typical value 0.08 for silt loam (*Johnson*, 1969 USGS), which was first suggested by *Yeh et al* (1998, JGR) in Illinois water balance studies, and later followed by *Eltahir and Yeh* (1999), *Seveviratne et al* (2004), and GRACE validation studies in Illinois by *Yeh and Famiglietti* (2006, 2008), *Rodell et al* (2007), and many others.

Main Research Questions



Theoretical Background

Groundw Water balance

$$\frac{dS}{dt} = I - Q_{gw}$$

Recession period

$$\longrightarrow \frac{dS}{dt} = -Q_{gw}$$

Q_{gw} : baseflow
 S : GW storage
 I : GW recharge
 and evaporation

Storage-Discharge Relation

$$Q_{gw} = mS^n$$

$b \geq 2$ not considered in
 this study

Brutsaert and Nieber (1977, WRR)

$$-\frac{dQ_{gw}}{dt} = aQ_{gw}^b$$

$$\begin{cases} n = \frac{1}{2-b} \\ m = [a(2-b)]^{\frac{1}{2-b}} \end{cases}$$

$b=1$, then $n=1$ &
 $m = a = 1/K$

Relations between Storage, Baseflow, and Water Table Depth

$$S = n_e (Z_0 - Z_{gw}) \quad Q_{gw} = k_0 (Z_0 - Z_{gw})$$

Z_{gw} : water table depth
 Z_0 : a threshold depth
 n_e : specific yield

Estimation of n_e

$$n_e = \frac{k_0^{2-b}}{(2-b)a^{2-b}}$$

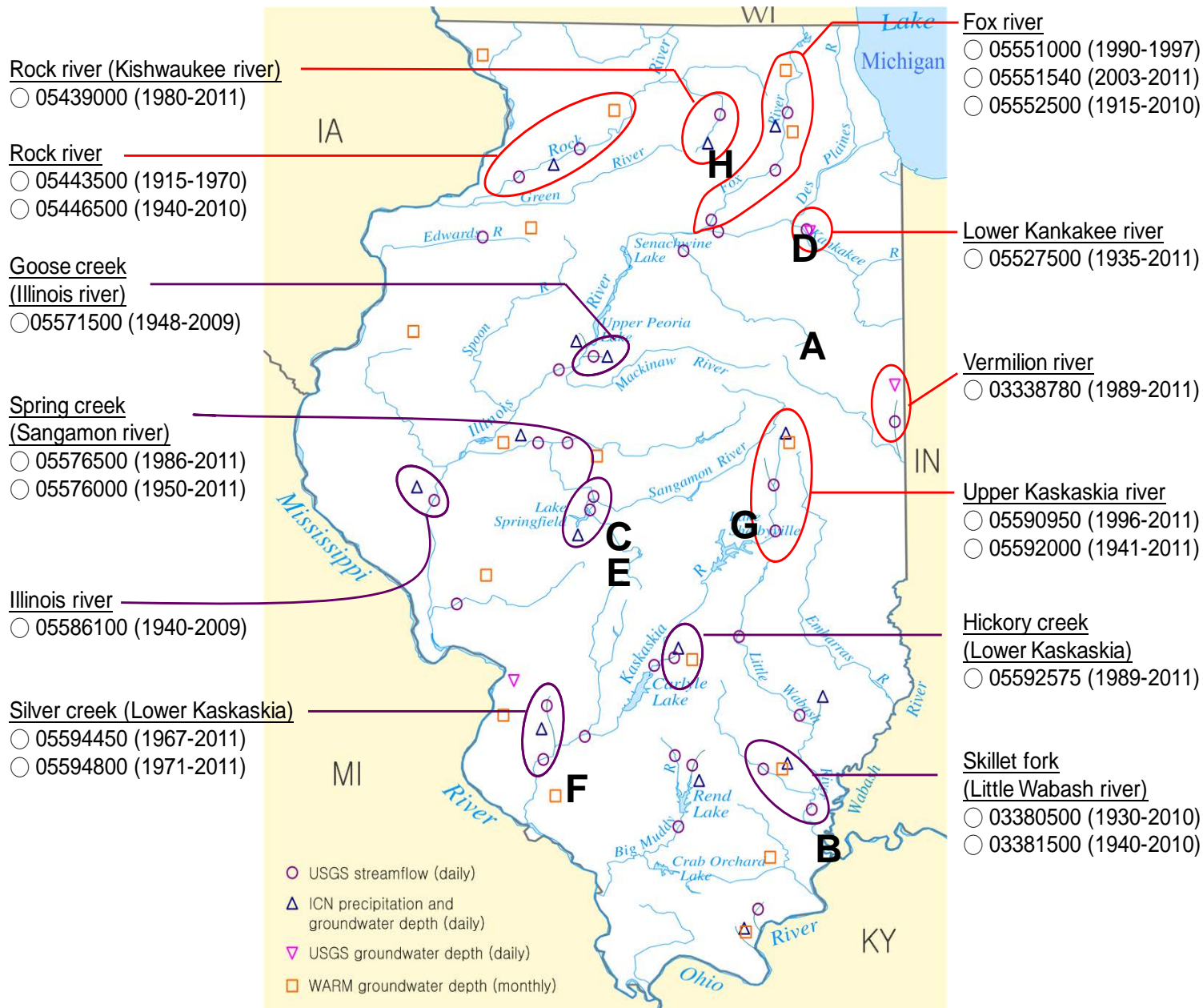
linear
 \longrightarrow
 $b=1$

$$n_e = k_0 K$$

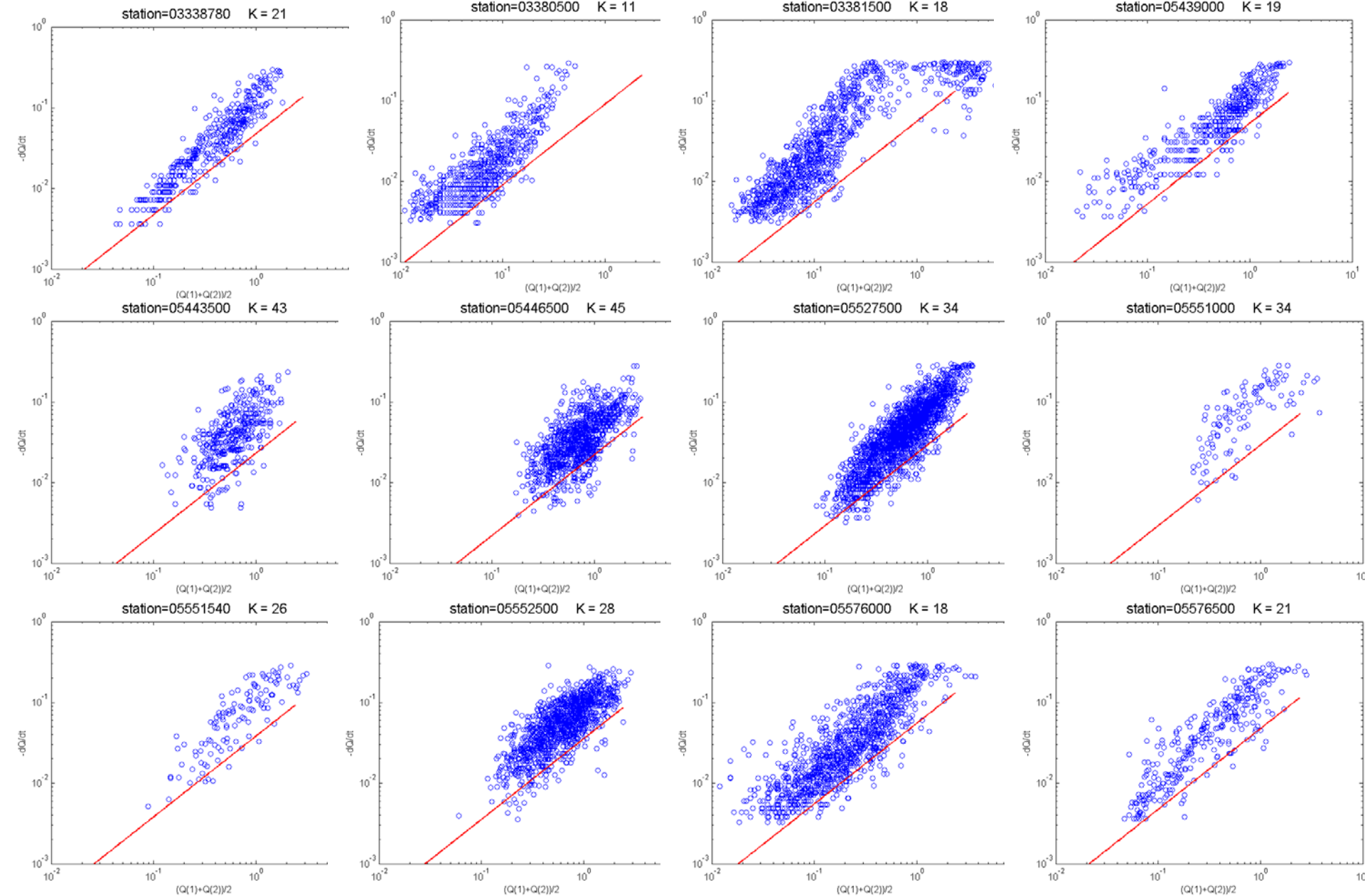
Procedures and Assumptions

- Streamflow recession data screening criteria followed exactly *Brutsaert and Nieber* (1977) and *Brutsaert* (2008)'s "lower envelop method" (by excluding 5% of the data below the slope line)
- Both linear and nonlinear analysis; using each single station of water table depth (WTD) measurements rather than state-average; long-term daily data of baseflow and WTD are used
- All analyses divided into warm (Apr.-Sep.) and cold season (Oct. – Mar.) to identify the influences of additional fluxes: evaporation (warm season) and groundwater recharge (cold season)
- Single-valued aquifer storage-discharge relation – No hysteresis
- Specific yield (n_e) estimated from only the recession period, not rising limb of hydrograph
- WTD – baseflow delay effect is not considered yet
- Spatial heterogeneity of water table depth (WTD) not considered – assuming each observed WTD data is representative for the catchment it is located

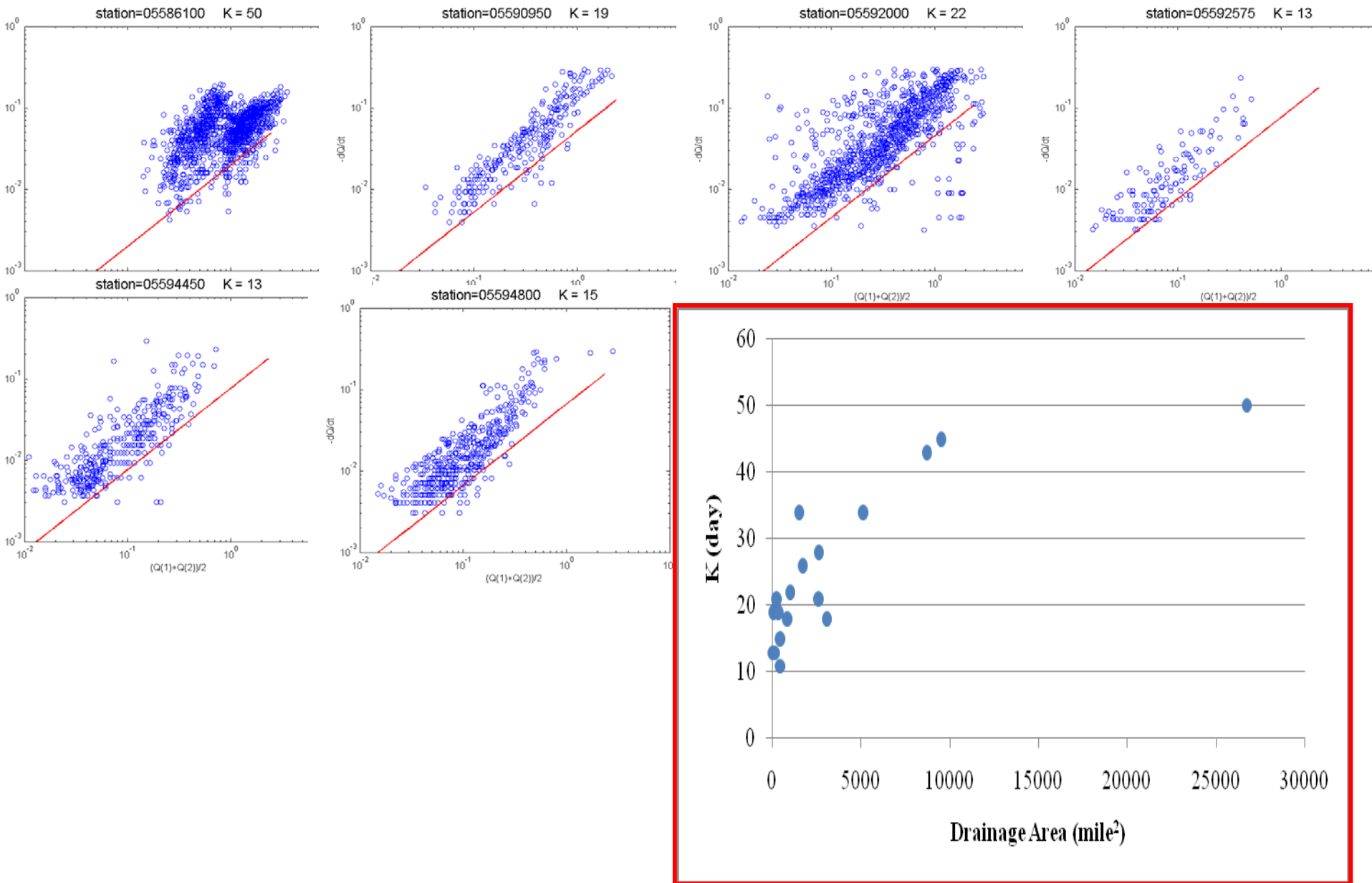
Illinois Water Table Depth and Streamflow Data



Recession Slope Curve (RSC) Analysis: Lower Envelope Method



Recession Slope Curve (RSC) Analysis: Lower Envelope Method

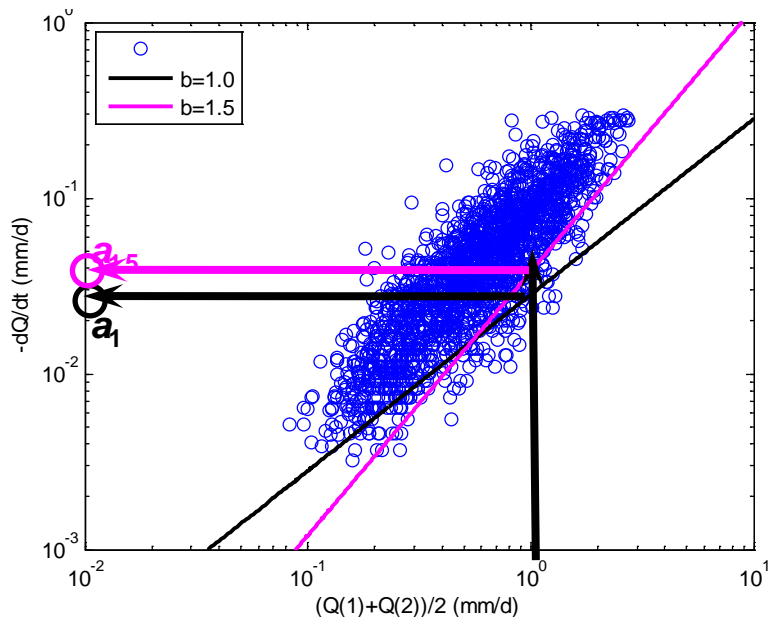


Uncertainty on procedures of estimating a and b

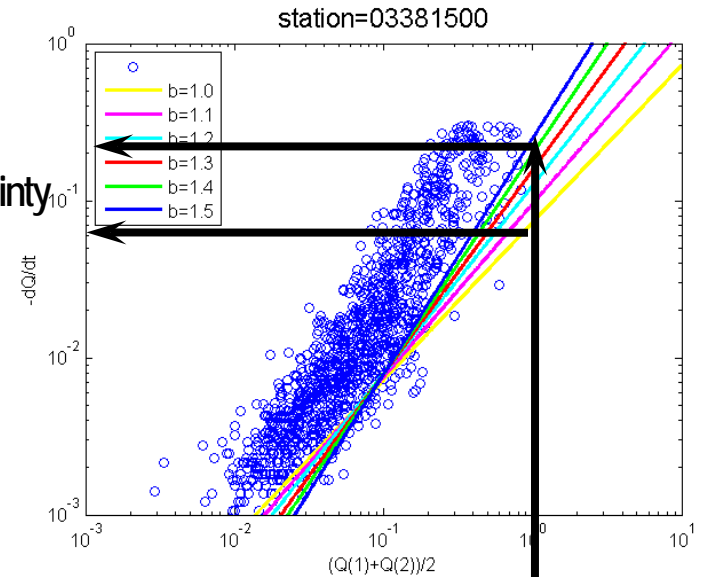
The value of a corresponds to the value of vertical axis when $Q_{gw} = 1$:

$$-\frac{dQ_{gw}}{dt} = aQ_{gw}^b$$

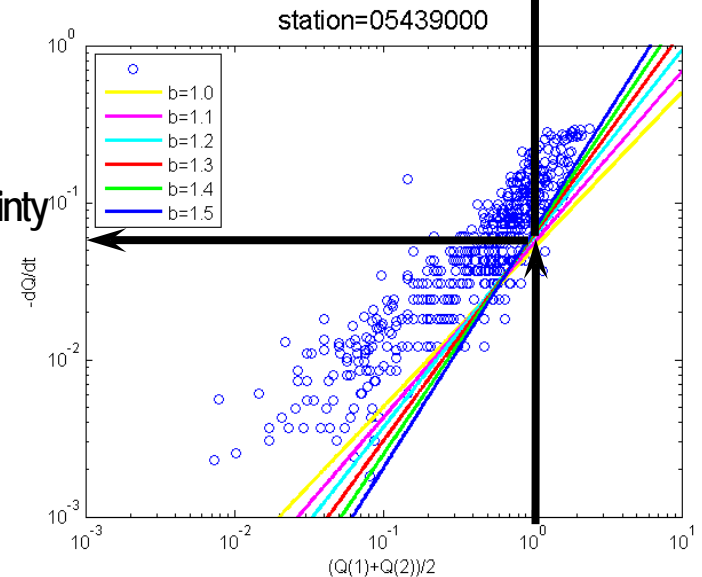
On the uncertainty of a , e.g, *Rupp and Selker (2006)*; *Szilagyi et al. (2007)*; *Palmroth et al. (2010)*

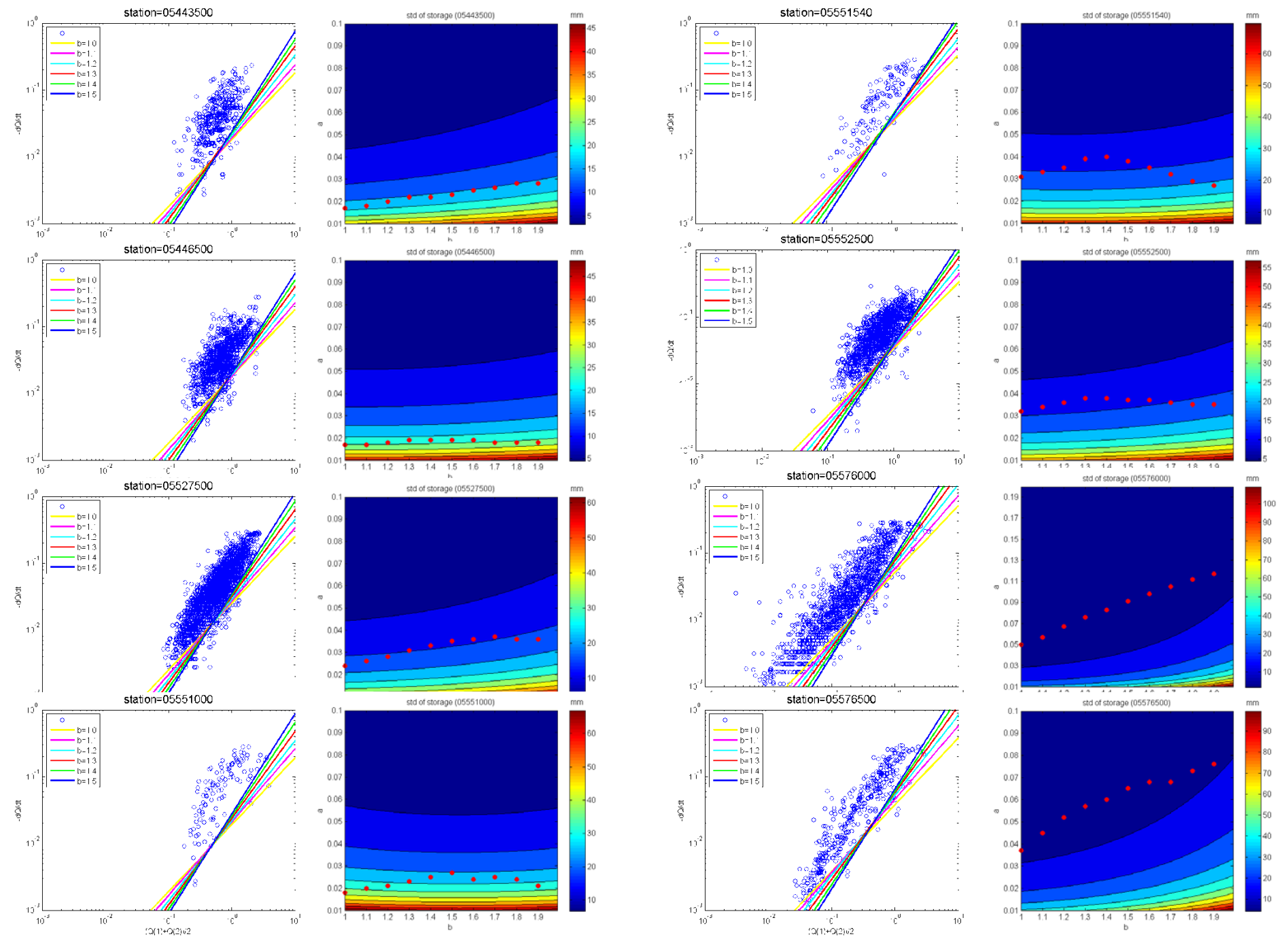


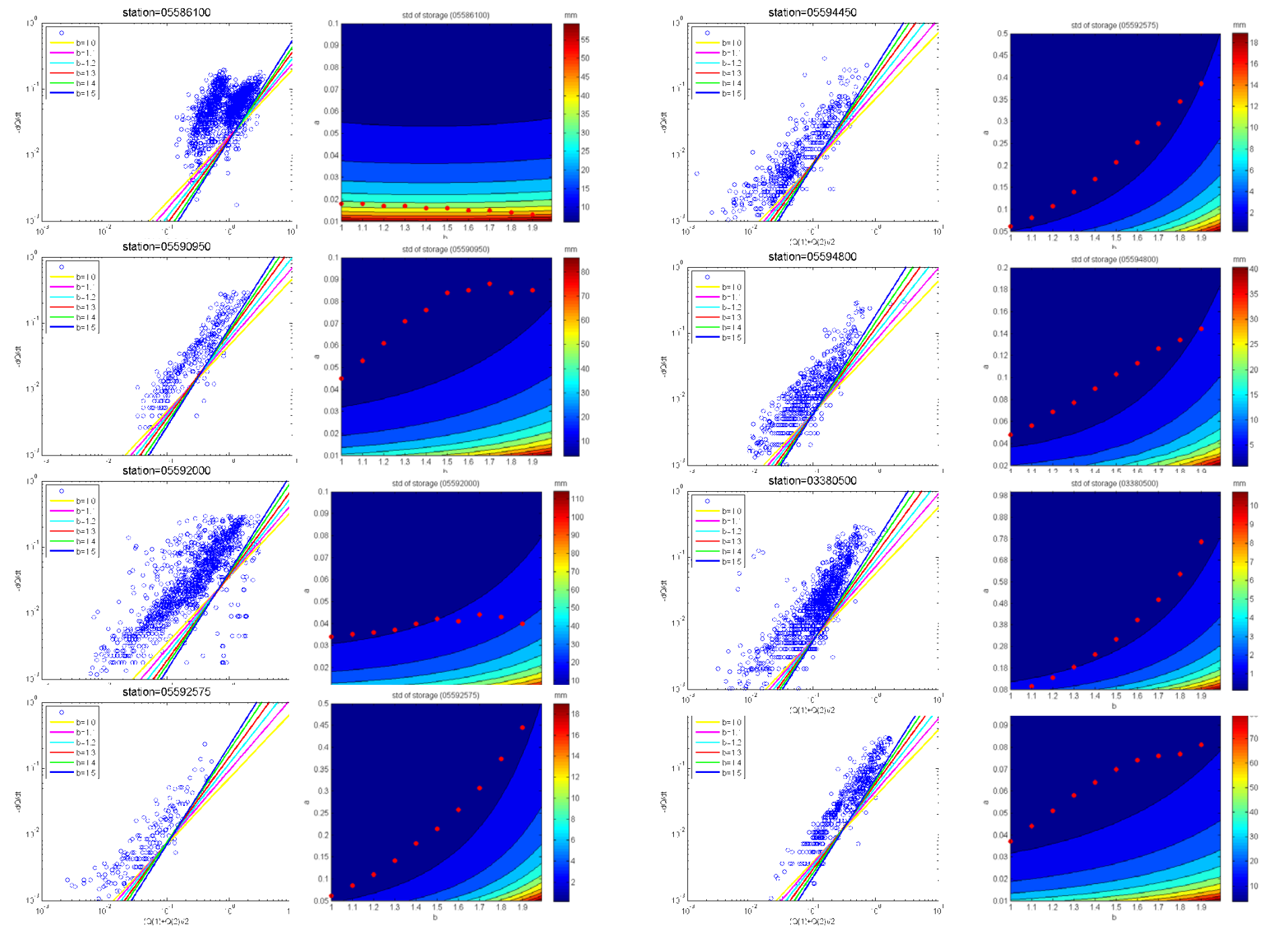
Large Uncertainty

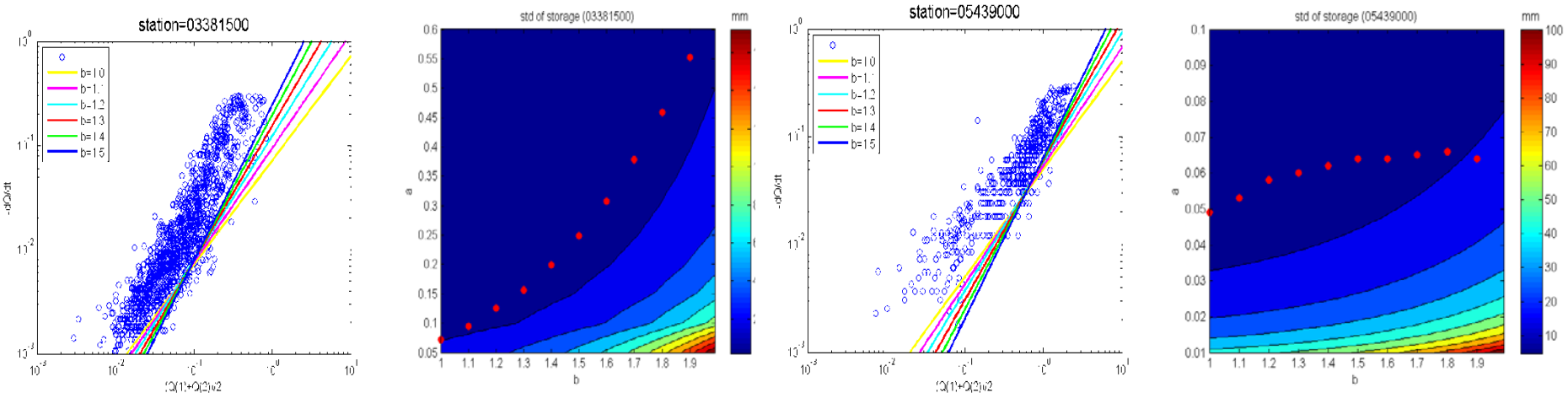


Small Uncertainty

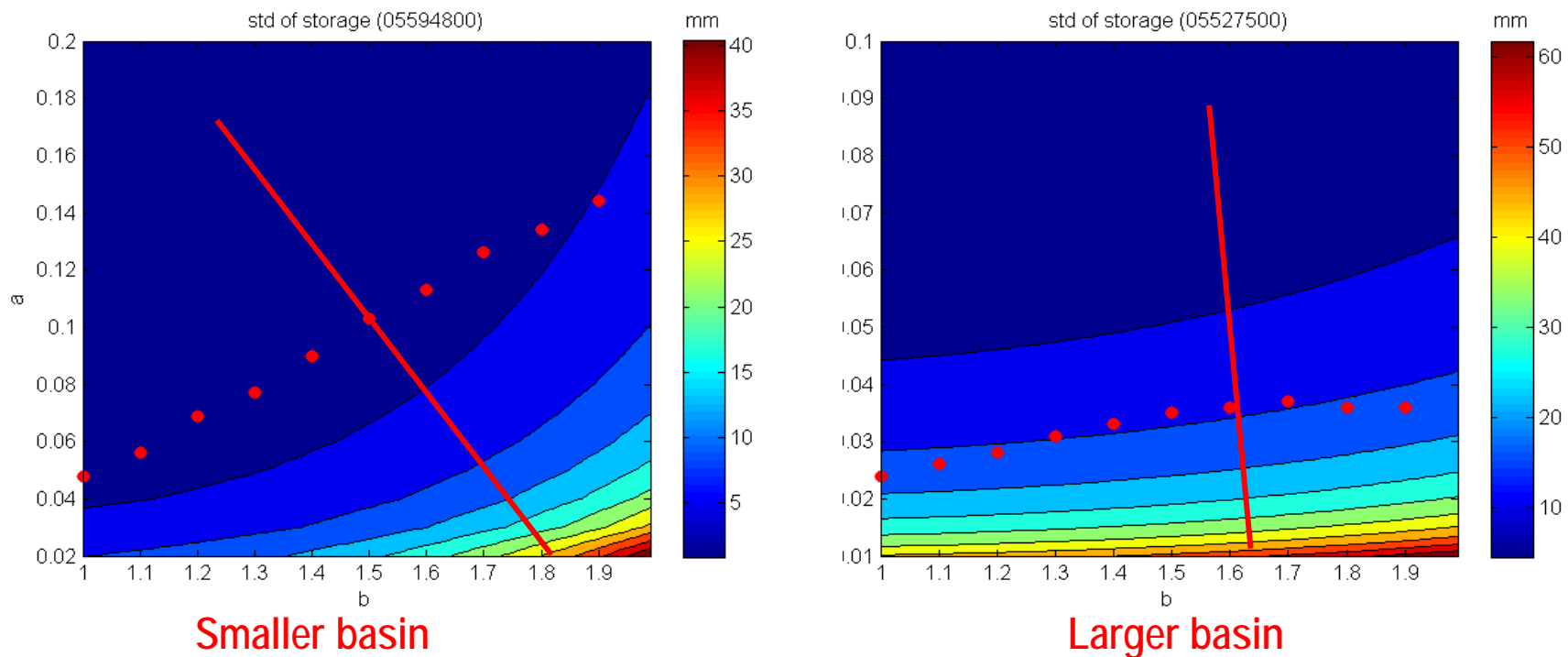






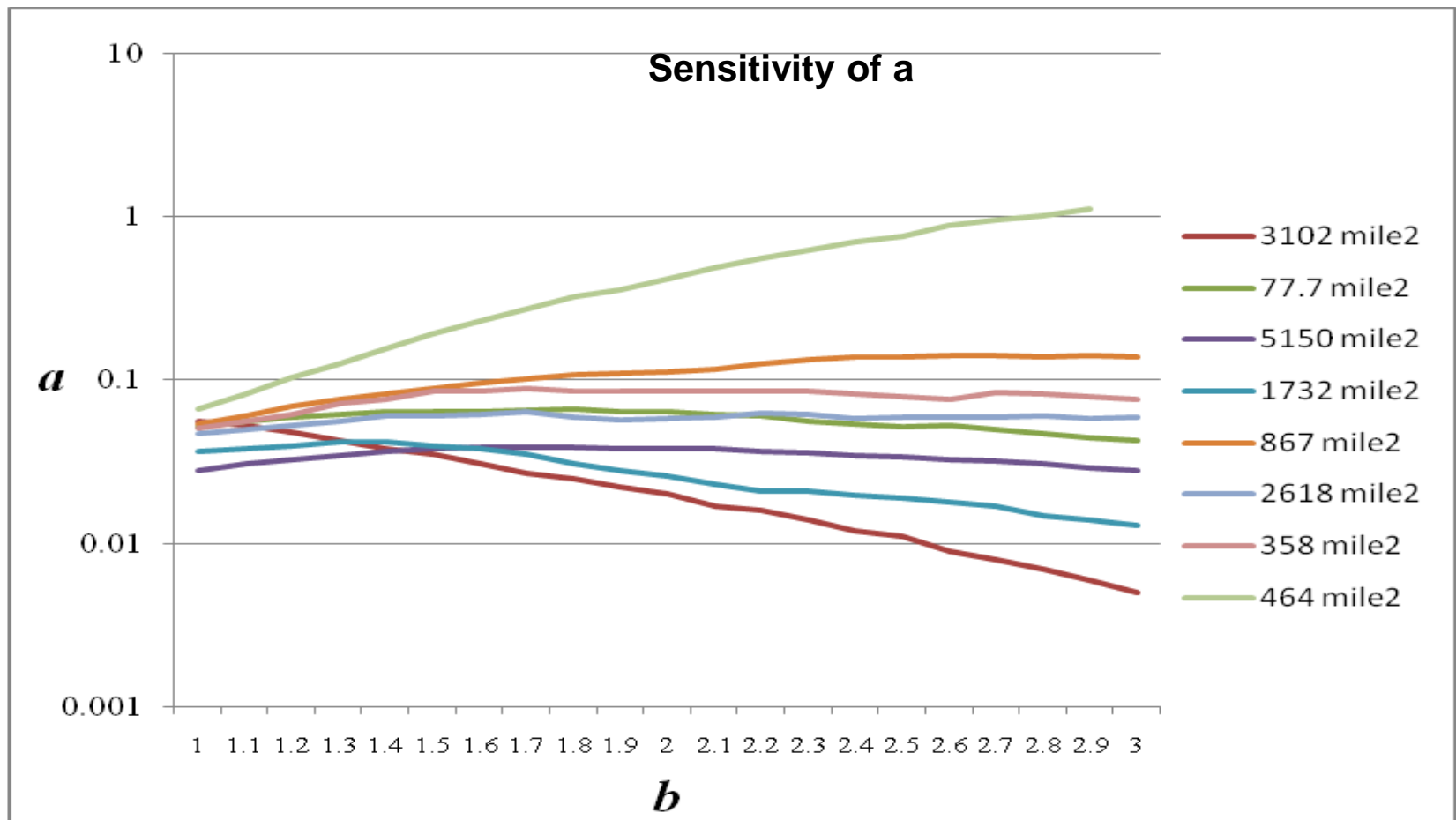


General Tendency of Sensitivity Analysis



Recession Slope Curve (RSC) Analysis: Lower Envelope Method

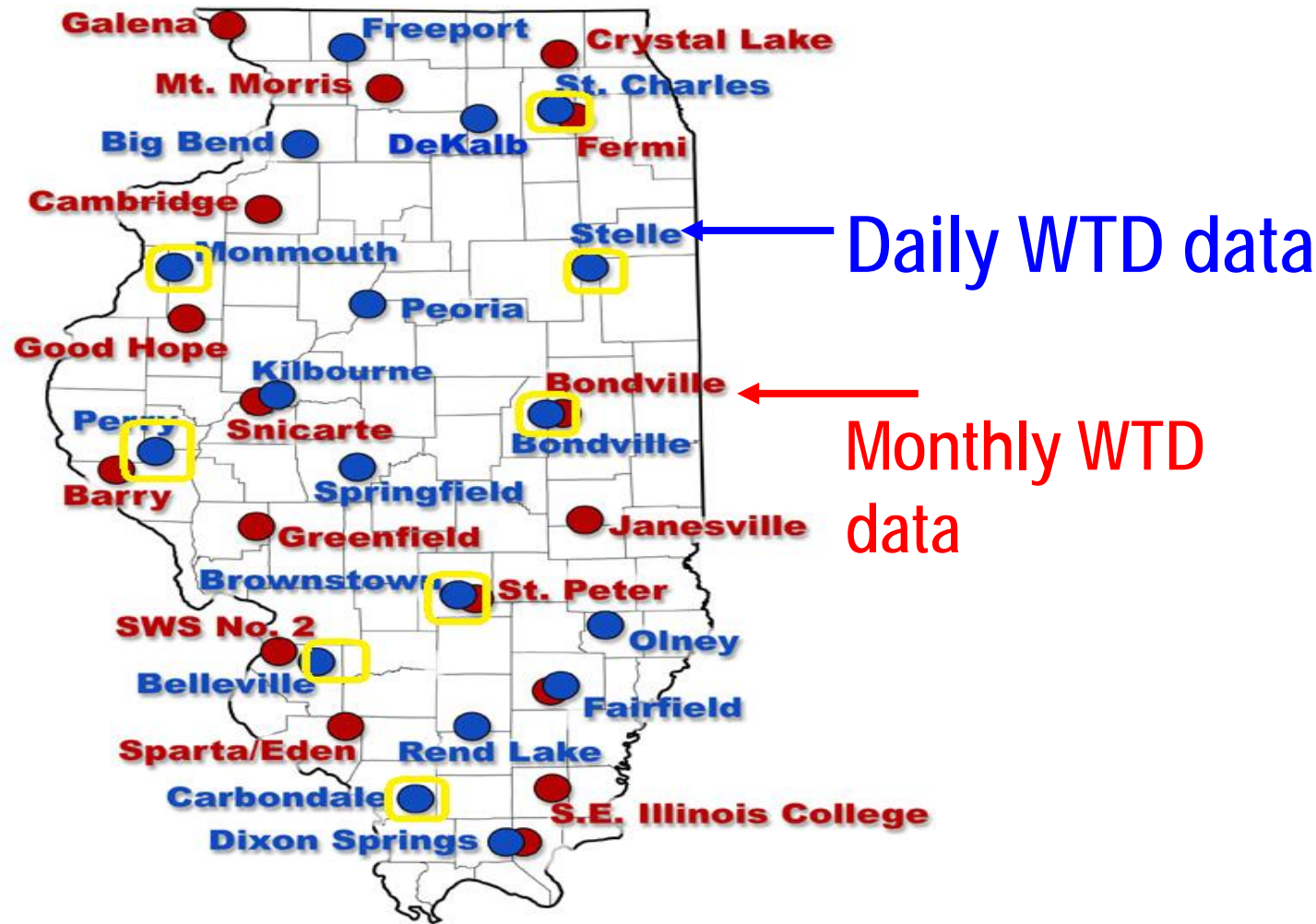
Nonlinear Analysis



Illinois Data: ISWS Daily Water Table Depth

ISWS (Illinois State Water Survey) ICN(Illinois Climate Network) wells -

Daily and Monthly measurement data of 17 wells for shallow groundwater from 1997-2010 , but the period available is different in different wells.



Daily WTD Data

Daily WTD data from Illinois State Water Survey (ISWS) from 1997 to 2009.

	USGS daily Streamflow				ISWS daily WTD					
	lat	lon		Distance (km)	period	lat	lon	mean (m)	std (m)	cv
05527500	41° 20'	88° 11'	A	1.3	2004-2009	41° 20'	88° 10'	0.963	0.657	0.682
03381500	38° 03'	88° 09'	B	41.9	2004-2009	38° 22'	88° 23'	1.020	0.661	0.648
05576500	39° 50'	89° 32'	C	14.7	2004-2009	39° 43'	89° 36'	1.737	0.914	0.526
05551540	41° 44'	88° 20'	D	19.1	2003-2009	41° 54'	88° 21'	7.705	1.157	0.150
05576000	39° 44'	89° 34'	E	4.4	2004-2009	39° 43'	89° 36'	1.737	0.914	0.526
05594800	38° 24'	89° 52'	F	12.8	2000-2009	38° 31'	89° 50'	1.686	1.42	0.842
05590950	39° 42'	88° 23'	G	38.9	2002-2009	40° 03'	88° 22'	1.382	0.691	0.500
05439000	41° 55'	88° 45'	H	12.5	1997-2009	41° 50'	88° 51'	1.039	0.681	0.655

Water Table Depth vs Baseflow: Threshold

Based on monthly observations in Illinois (*Yeh and Eltahir, 2005*), the following threshold relationship between baseflow and WTD can be assumed:

$$Q_{gw} = k_0 (z_0 - z_{gw}) \quad \text{if } 0 \leq z_{gw} \leq z_0$$

$$Q_{gw} = 0 \quad \text{if } z_{gw} \geq z_0$$

Q_{gw} : baseflow
 k_0 : outflow constant
 z_{gw} : water table depth (WTD)
 z_0 : threshold WTD

Parameters K and d_0 can be fit optimally by varying d_0 and K simultaneously within a reasonable range until the optimal values are found that minimize the sum of absolute error.

15 JUNE 2005

YEH AND ELTAHIR

$$A = A_1 + A_2 + A_3 = \int_{d_{gw}=0}^{d_{gw}=d_3} f(d_{gw}) d(d_{gw}) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \left[\frac{(\alpha-1)!}{\lambda^\alpha} - e^{-\lambda d_3} \sum_{k=0}^{\alpha-1} \frac{(\alpha-1)!}{k!} \frac{d_3^k}{\lambda^{\alpha-k}} \right]$$

$$A_1 = \int_{d_{gw}=0}^{d_{gw}=d_1} f(d_{gw}) d(d_{gw}) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \left[\frac{(\alpha-1)!}{\lambda^\alpha} - e^{-\lambda d_1} \sum_{k=0}^{\alpha-1} \frac{(\alpha-1)!}{k!} \frac{d_1^k}{\lambda^{\alpha-k}} \right]$$

$$A_2 = \int_{d_{gw}=0}^{d_{gw}=d_2} f(d_{gw}) d(d_{gw}) - \int_{d_{gw}=0}^{d_{gw}=d_1} f(d_{gw}) d(d_{gw});$$

$$A_3 = \int_{d_{gw}=0}^{d_{gw}=d_3} f(d_{gw}) d(d_{gw}) - \int_{d_{gw}=0}^{d_{gw}=d_2} f(d_{gw}) d(d_{gw}).$$

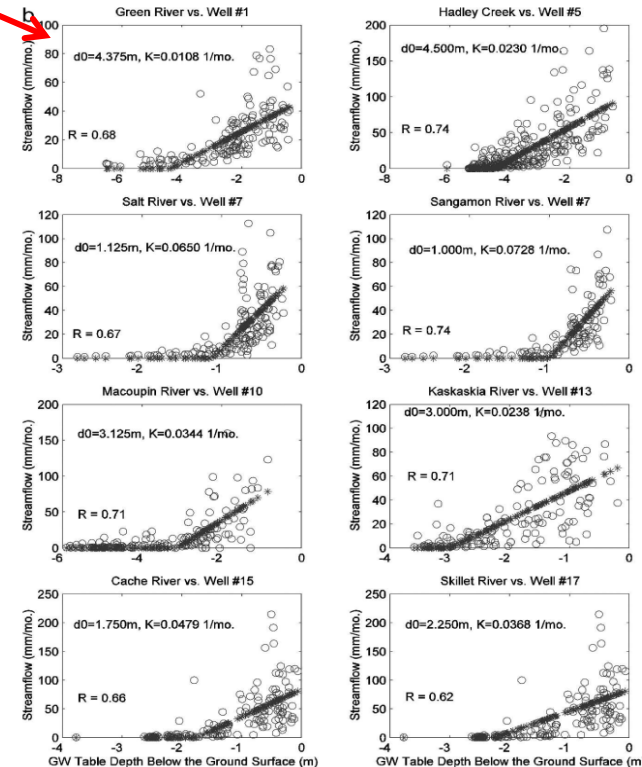
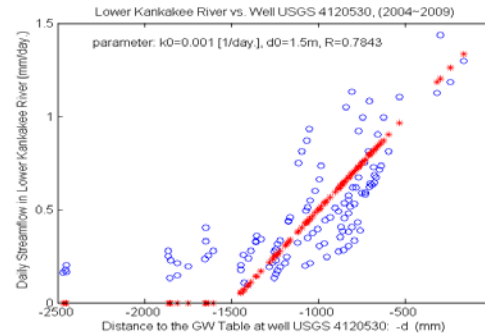
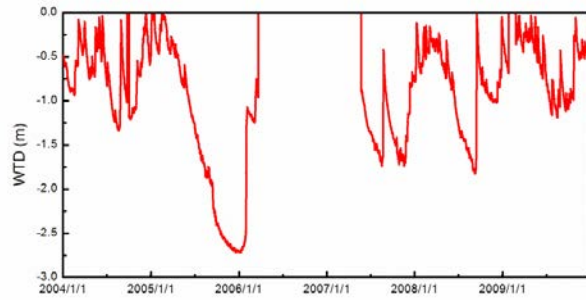


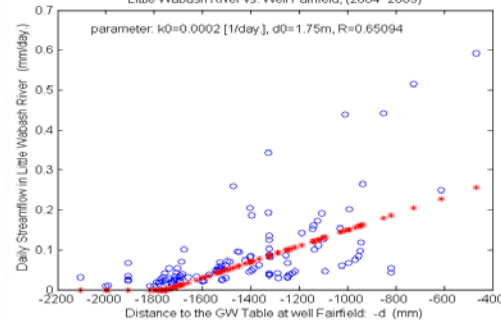
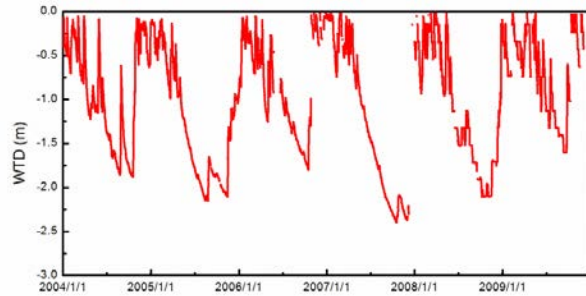
FIG. 2. (Continued)

A



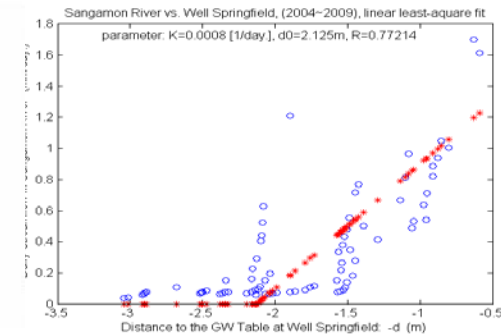
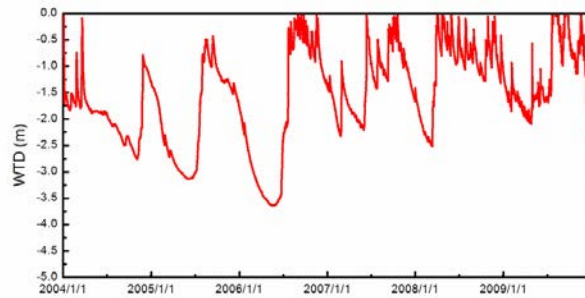
$k_0=0.01/\text{day}$
 $d_0=1.5\text{m}$
 $R=0.78$

B



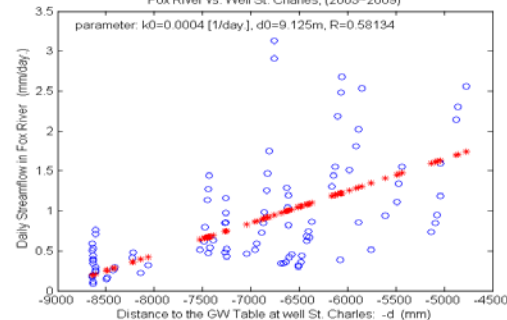
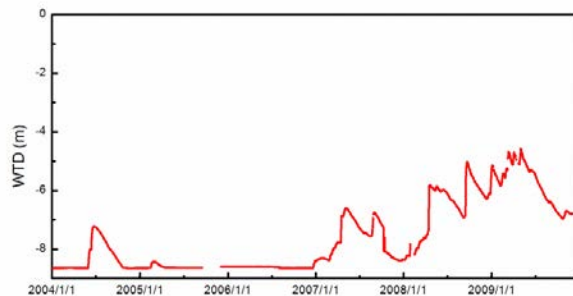
$k_0=0.0002/\text{day}$
 $d_0=1.75\text{m}$
 $R=0.65$

C



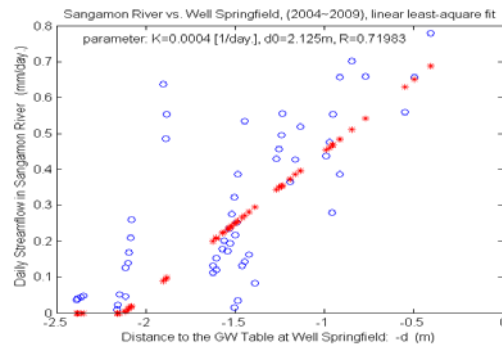
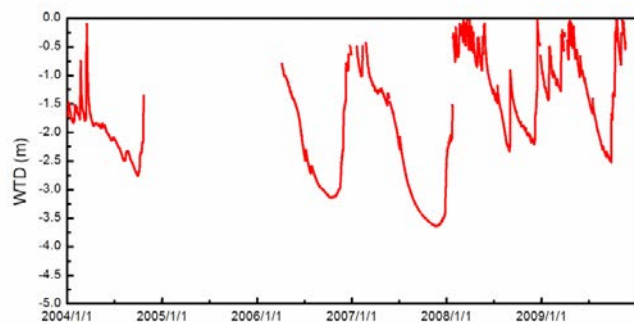
$k_0=0.0008/\text{day}$
 $d_0=2.125\text{m}$
 $R=0.77$

D



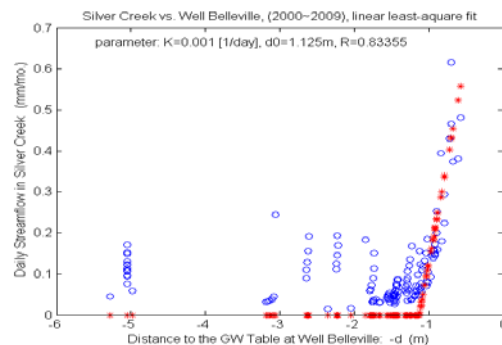
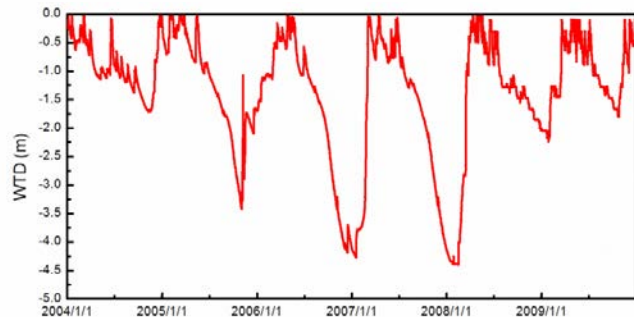
$k_0=0.0004/\text{day}$
 $d_0=9.125\text{m}$
 $R=0.58$

E



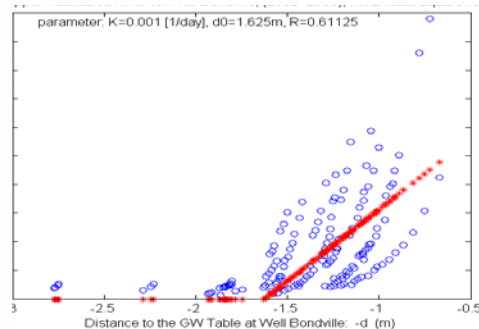
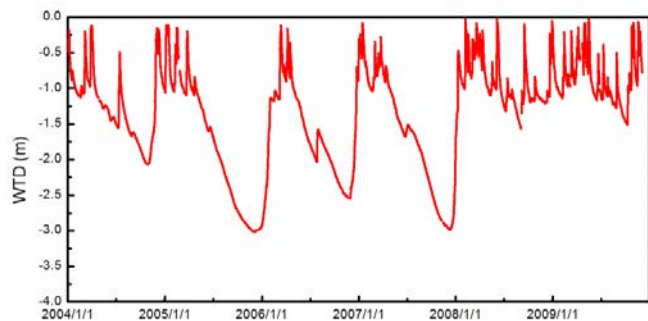
$k_0=0.0004/\text{day}$
 $d_0=2.125\text{m}$
 $R=0.72$

F



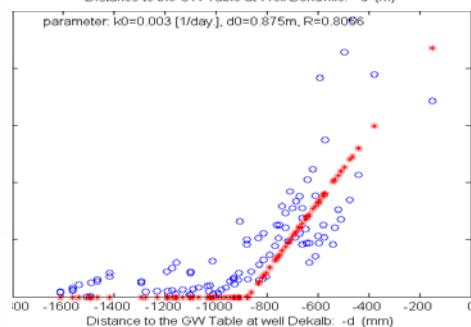
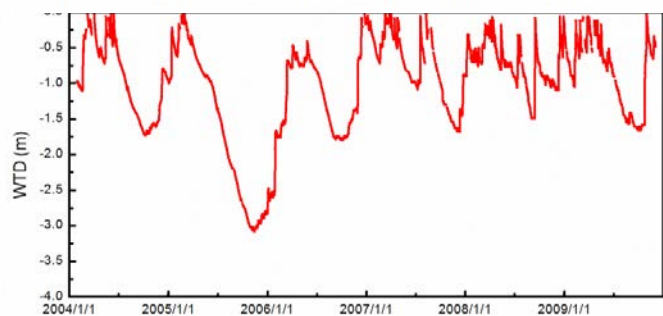
$k_0=0.001/\text{day}$
 $d_0=1.125\text{m}$
 $R=0.83$

G



$k_0=0.001/\text{day}$
 $d_0=1.625\text{m}$
 $R=0.61$

H



$k_0=0.003/\text{day}$
 $d_0=0.875\text{m}$
 $R=0.81$

Summary of Baseflow vs WTD (Threshold Linear)

Pair	K (day)		k_o (1/day)		R (Correlation)	
	Apr-Sep	Oct-Mar	Apr-Sep	Oct-Mar	Apr-Sep	Oct-Mar
A	40	53	0.0016	0.0010	0.651	0.565
B	14	20	0.0002	0.0020	0.806	0.728
C	26	31	0.0008	0.0002	0.766	0.668
D	31	40	0.0004	0.0002	0.581	0.681
E	20	29	0.0004	0.0020	0.720	0.280
F	16	21	0.0010	0.0002	0.772	0.335
G	22	28	0.0010	0.0020	0.611	0.710
H	20	24	0.0030	0.0016	0.834	0.506

Estimation of specific yield n_e

Brutsaert and Nieber (1977)

$$n_e = \frac{k_0^{2-b}}{(2-b)a^{2-b}} \xrightarrow[\text{b=1}]{\text{linear}} n_e = k_0 K$$

Brutsaert and Lopez (1998)

$$n_e = 1.9688(a_1 a_3)^{-0.5} (DA)^{-1}$$

Mass conservation equation

$$Q_{gw} = -n_e \frac{dz_{gw}}{dt} A$$

n_e : specific yield

k_0 : coefficient of threshold model (1/day)

K : residence time (day)

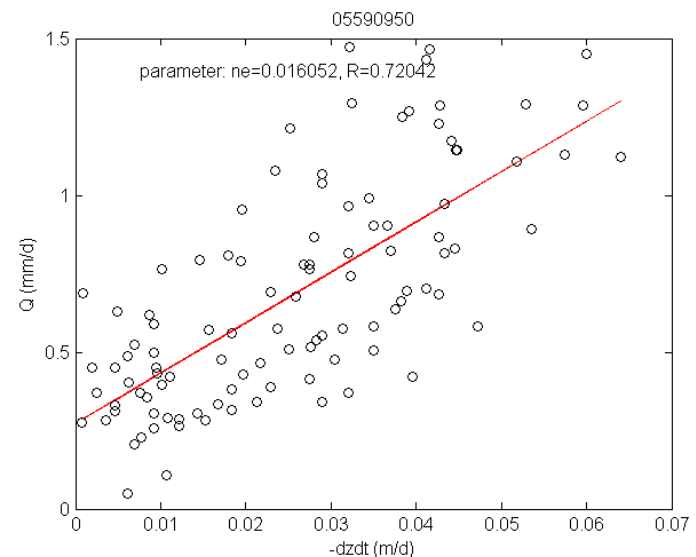
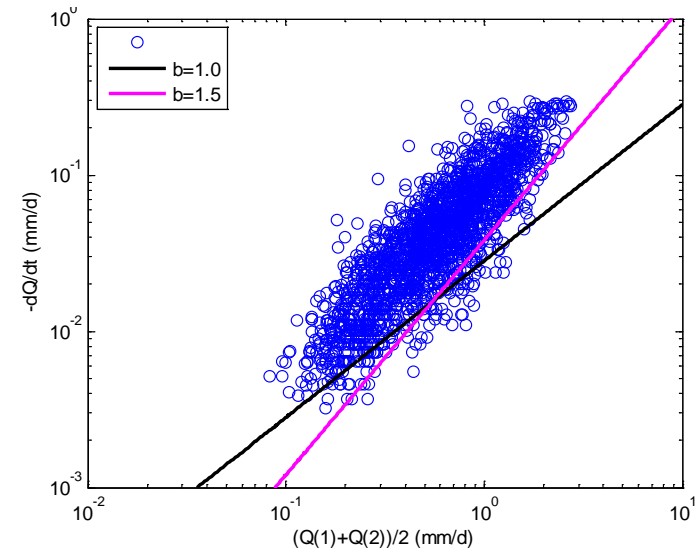
a_1 : the value of a when $b=1$ (day⁻¹)

a_3 : the value of a when $b=3$ (day*mm⁻⁶)

D : mean depth (taken as the deepest WTD)

A : basin area (km²)

z : water table depth (m)



Comparison of n_e Estimates

Correlation from

dz/dt

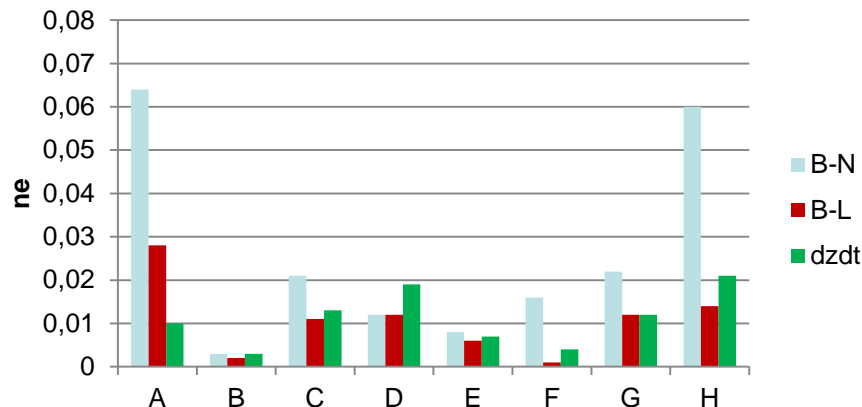


linear

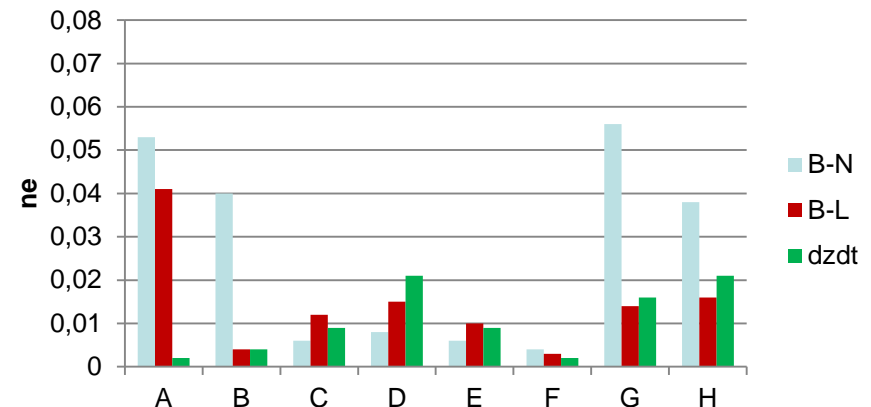
	B-N (1977)		B-L (1998)		dz/dt		R	
	Apr-Sep	Oct-Mar	Apr-Sep	Oct-Mar	Apr-Sep	Oct-Mar	Apr-Sep	Oct-Mar
A	0.064	0.053	0.028	0.041	0.010	0.002	0.251	0.069
B	0.003	0.04	0.002	0.004	0.003	0.004	0.598	0.098
C	0.021	0.006	0.011	0.012	0.013	0.009	0.407	0.586
D	0.012	0.008	0.012	0.015	0.019	0.021	0.217	0.441
E	0.008	0.006	0.006	0.01	0.007	0.009	0.502	0.554
F	0.016	0.004	0.001	0.003	0.004	0.002	0.694	0.333
G	0.022	0.056	0.012	0.014	0.012	0.016	0.479	0.720
H	0.060	0.038	0.014	0.016	0.021	0.021	0.599	0.733

basin D: WTD ~8m

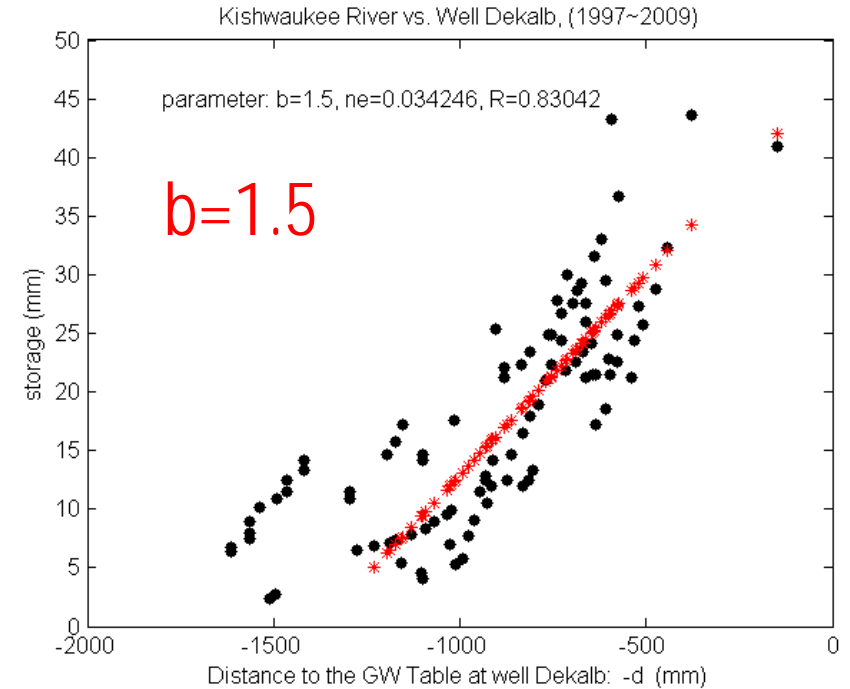
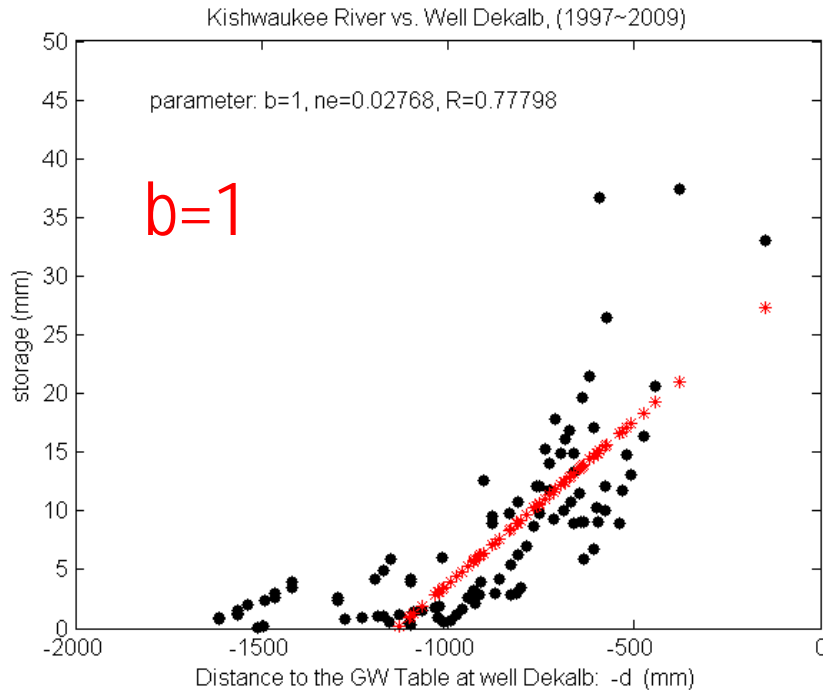
Apr-Sep



Oct-Mar



Estimation of n_e (linear vs nonlinear)



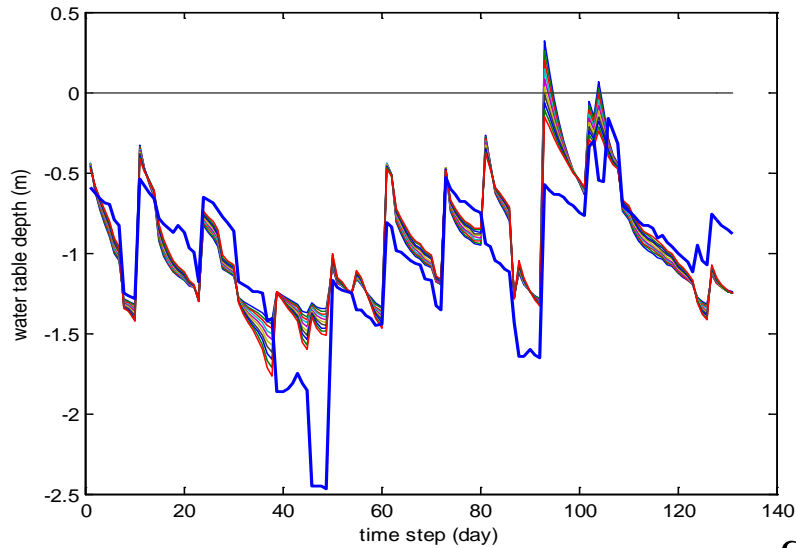
$$S = n_e (d_0 - d_{gw})$$

Summary of Optimal Parameters Identified

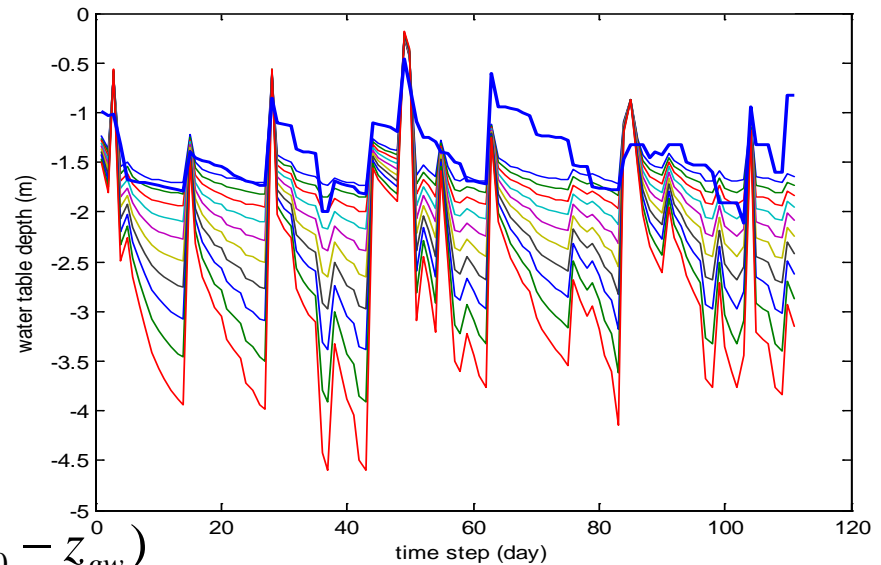
	Optimal a				Optimal b				Optimal ne			
	Apr-Sep		Oct-Mar		Apr-Sep		Oct-Mar		Apr-Sep		Oct-Mar	
	Based on R	Based on RMSE	Based on R	Based on RMSE	Based on R	Based on RMSE	Based on R	Based on RMSE	Based on R	Based on RMSE	Based on R	Based on RMSE
A	0.038	0.037	0.026	0.026	1.9	1.4	1.9	1.7	0.045	0.034	0.044	0.042
B	0.553	0.308	0.003	0.003	1.9	1.6	1.9	1.9	0.003	0.003	0.010	0.010
C	0.053	0.053	0.067	0.054	1.2	1.2	1.9	1.5	0.023	0.023	0.018	0.013
D	0.028	0.042	0.025	0.025	1.9	1.3	1.0	1.0	0.013	0.006	0.008	0.008
E	0.054	0.061	0.057	0.062	1.0	1.1	1.4	1.5	0.010	0.010	0.006	0.006
F	0.358	0.228	0.113	0.090	1.9	1.6	1.6	1.4	0.006	0.005	0.003	0.003
G	0.086	0.072	0.036	0.060	1.9	1.3	1.0	1.7	0.021	0.012	0.045	0.034
H	0.064	0.059	0.054	0.047	1.6	1.2	1.9	1.2	0.038	0.028	0.046	0.029

Estimated Water Table Depth

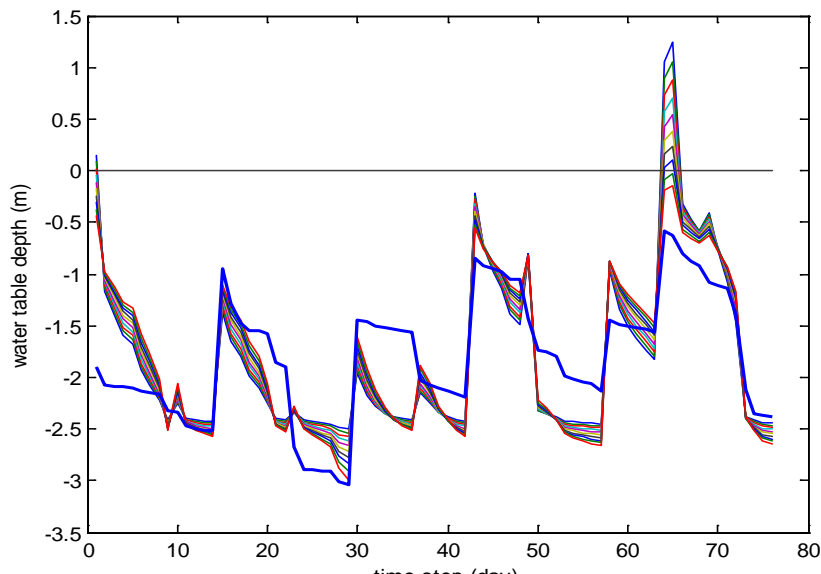
A



B

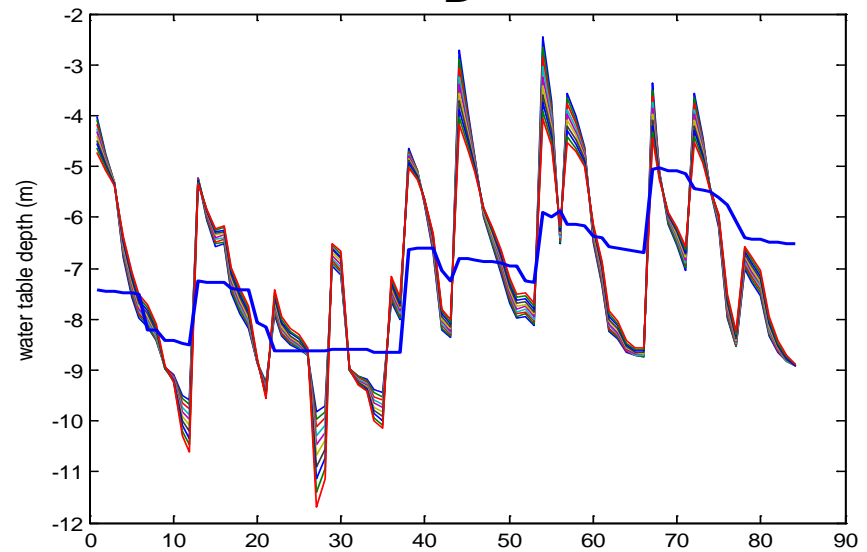


C



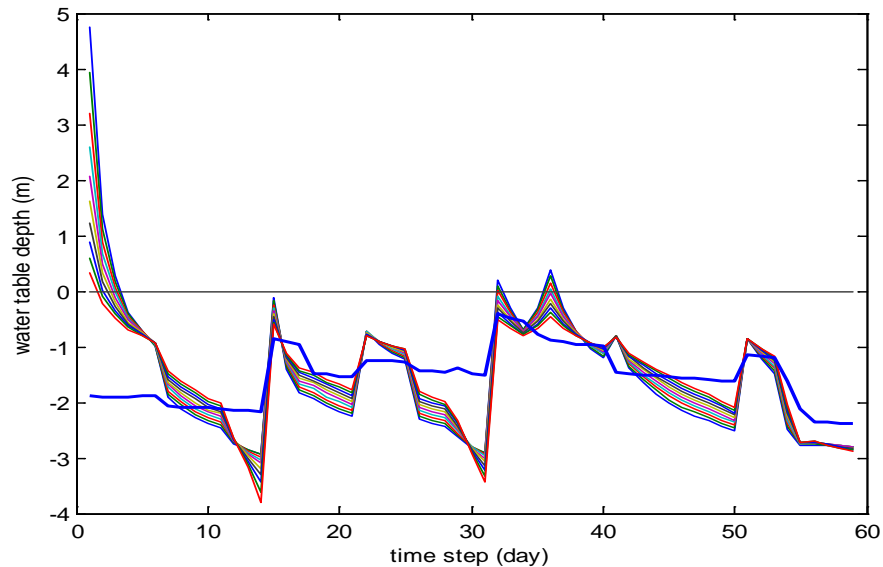
$$S = n_e (z_0 - z_{gw})$$

D

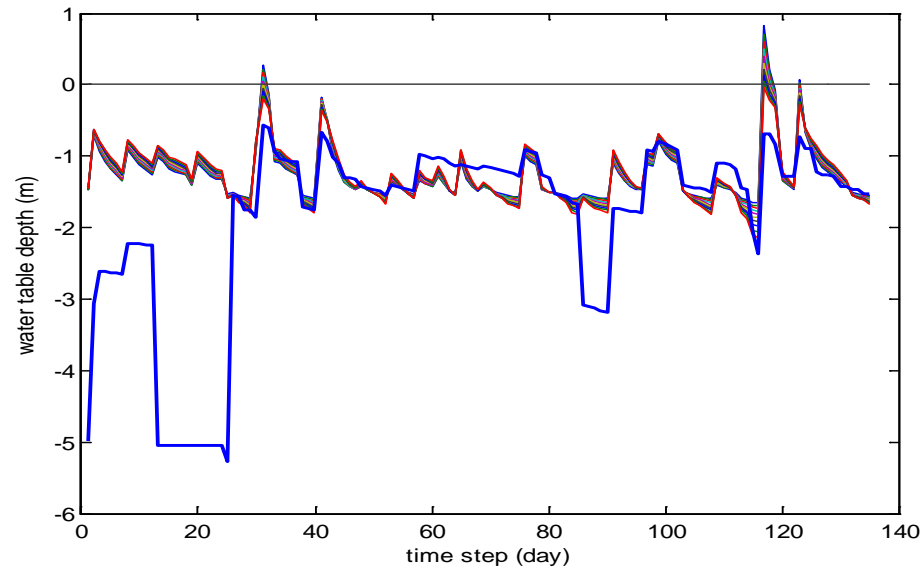


Estimated Water Table Depth

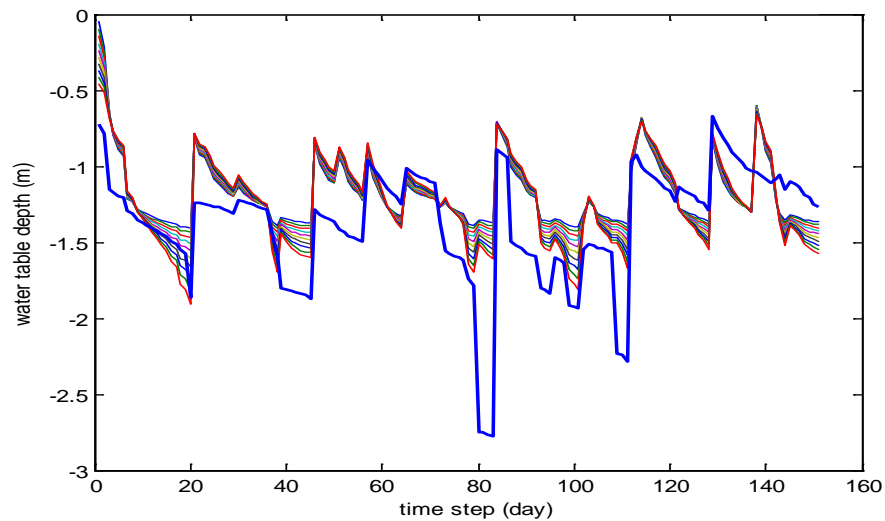
E



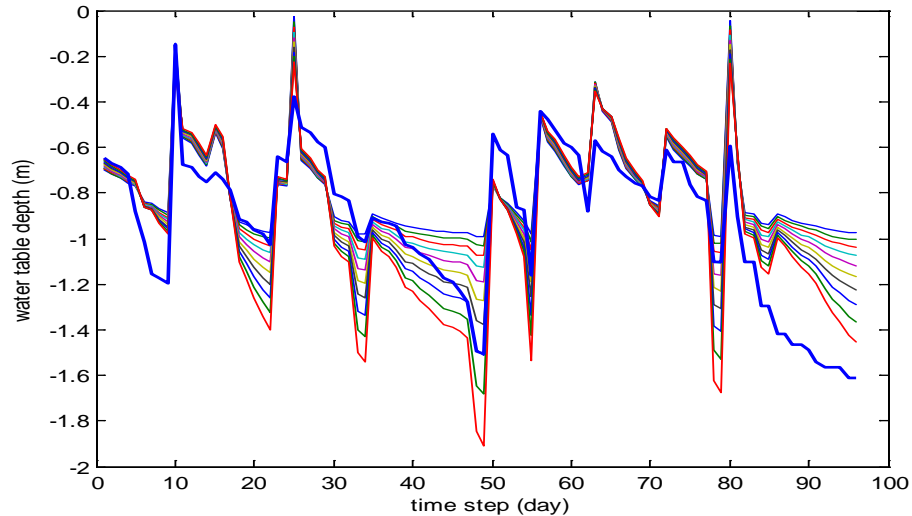
F



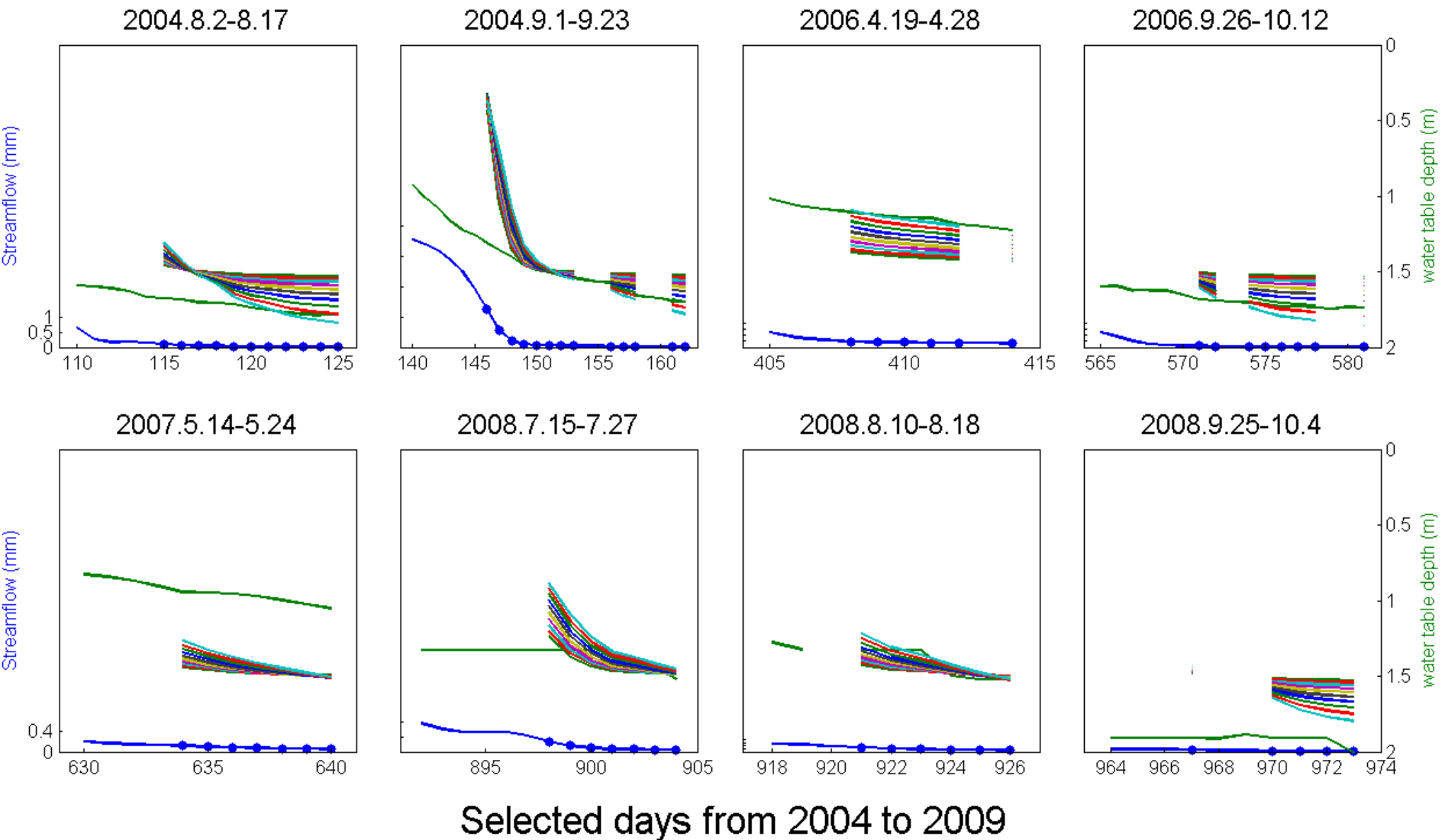
G



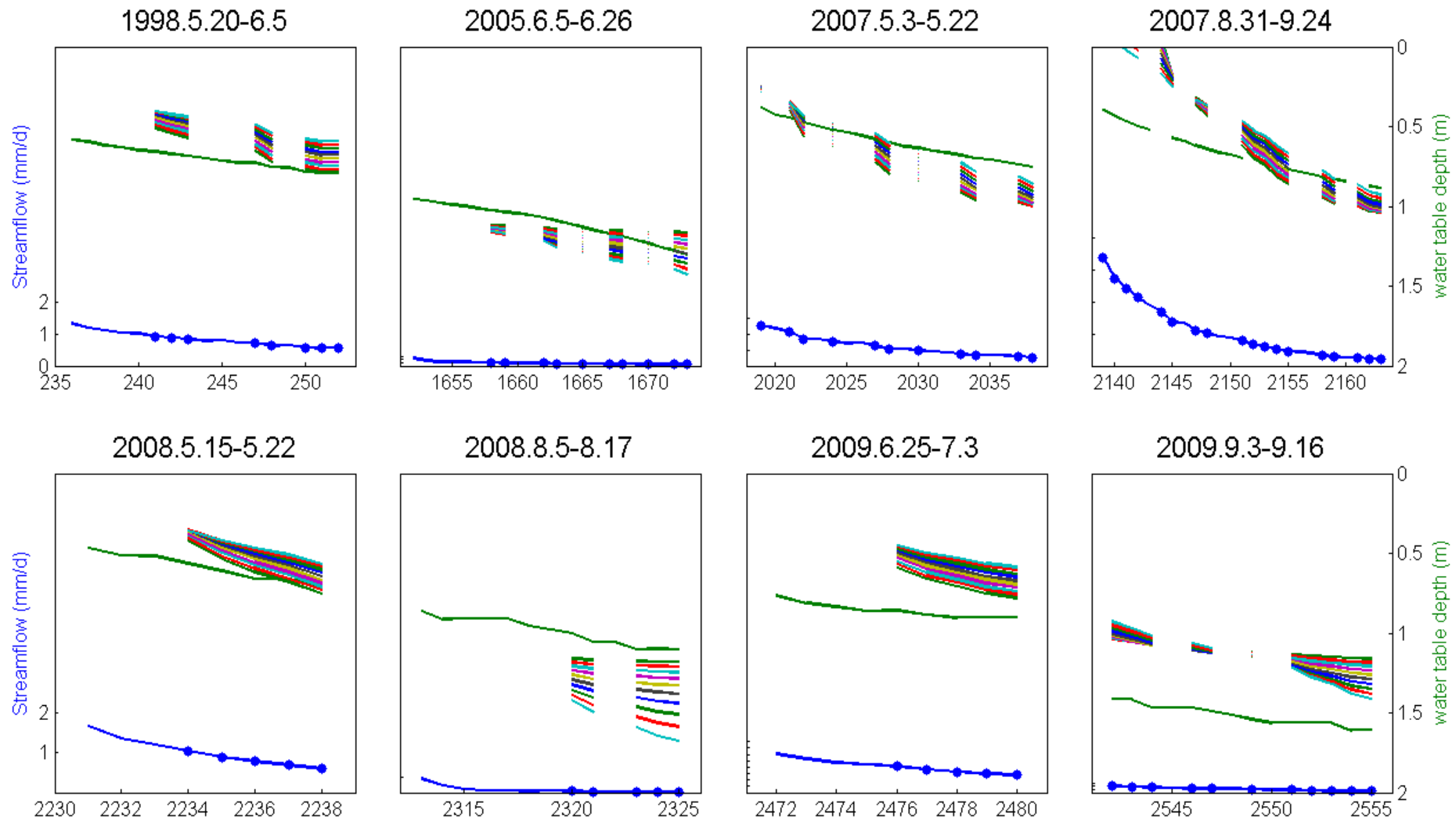
H



Comparison of Observed and Estimated Water Table (Pair B)



Comparison of Observed and Estimated Water Table (Pair H)



Selected days from 1997 to 2009

Summary of Part II Study

From the linear analysis, the (aquifer) residence time increases with basin size. It ranges from 11-55 days for warm seasons and 18-77 days for cold season. Warm season always shorter than cold season due to evaporation.

From the nonlinear analysis, the placement of recession slope curve remains somewhat ambiguous for certain basins. However, it is encouraging that the estimated storage variance is not sensitive to the nonlinearity exponent b particularly for the large basins.

Due to above, the estimated specific yield does not change much regardless of b , and it is also close to the estimates from another two independent approaches.

Water table is not connected to baseflow when it drops under a threshold value due to evaporation in warm season. Under such conditions, the predicted aquifer storage based on identified baseflow recession parameters (a & b) cannot predict water table variations.

Brustaert (2008)'s argument that specific yield (n_e) is smaller than the typically values 0.08 for silt loam soils in Illinois was conformed here. Estimated n_e in this study ranges from 0.003 - 0.046, all smaller than 0.08. However, when the state-average water table depth is used instead of point measurements, n_e can be much larger due to reduced variance of WTD.

Scale effect will be investigated. Day-to-day delay has to be accounted for better correlation.

Thank You